Take : Specific enthalpy of fusion of ice = 336 kJ/kg

Specific heat of water = 4.187 kJ/kg. [**Ans.** 4.97, 10.9°C, 91.3 kg]

15. A vapour compression refrigerator circulates 4.5 kg of NH_3 per hour. Condensation take place at 30°C and evaporation at – 15°C. There is no under-cooling of the refrigerant. The temperature after isentropic compression is 75°C and specific heat of superheated vapour is 2.82 kJ/kg K. Determine :

(*i*) Co-efficient of performance.

(*ii*) Ice produced in kg per hour in the evaporator from water at 20°C and ice at 0°C. Take : Enthalpy of fusion of ice = 336 kJ/kg, specific heat of water = 4.187 kJ/kg.

(*iii*) The effective swept volume of the compressor in m³/min.

Properties of ammonia :

Sat. temp. (K)	*Enthalpy (kJ/kg)*		*Entropy (kJ/kg K)*		*Volume (m³/kg)*	
	h_f	h_g	s_f	s_g	v_f	v_g
303	323.1	1469	1.204	4.984	0.00168	0.111
258	112.3	1426	0.457	5.549	0.00152	0.509

[**Ans.** 4.95, 682 kg/h, 2.2 m³/min]

15

Heat Transfer

15.1. Modes of heat transfer. 15.2. Heat transfer by conduction—Fourier's law of heat conduction—Thermal conductivity of materials—Thermal resistance (R_{th})—General heat conduction equation in Cartesian coordinates—Heat conduction through plane and composite walls—Heat conduction through a plane wall—Heat conduction through a composite wall—The overall heat-transfer coefficient—Heat conduction through hollow and composite cylinders—Heat conduction through a hollow cylinder—Heat conduction through a composite cylinder—Heat conduction through hollow and composite spheres—Heat conduction through hollow sphere—Heat conduction through a composite sphere—Critical thickness of insulation—Insulation-General aspects—Critical thickness of insulation. 15.3. Heat transfer by convection. 15.4. Heat exchangers—Introduction—Types of heat exchangers—Heat exchanger analysis—Logarithmic mean temperature difference (LMTD)—Logarithmic mean temperature difference for "parallel-flow"—Logarithmic mean temperature difference for "counter-flow". 15.5. Heat transfer by radiation—Introduction—Surface emission properties—Absorptivity, reflectivity and transmissivity—Concept of a black body—The Stefan-Boltzmann law—Kirchhoff's law—Planck's law—Wien's displacement law—Intensity of radiation and Lambert's cosine law—Intensity of radiation—Lambert's cosine law—Radiation exchange between black bodies separated by a non-absorbing medium. Highlights—Objective Type Questions—Theoretical Questions—Unsolved Examples.

15.1. MODES OF HEAT TRANSFER

"Heat transfer" which is defined as the *transmission of energy from one region to another as a result of temperature gradient* takes place by the following *three modes :*

(*i*) Conduction ; (*ii*) Convection ; (*iii*) Radiation.

Heat transmission, in majority of real situations, occurs as a result of combinations of these modes of heat transfer. *Example :* The water in a boiler shell receives its heat from the fire-bed by conducted, convected and radiated heat from the fire to the shell, conducted heat through the shell and conducted and convected heat from the inner shell wall, to the water. *Heat always flows in the direction of lower temperature.*

The above three modes are similar in that a temperature differential must exist and the heat exchange is in the direction of decreasing temperature ; each method, however, has different controlling laws.

(*i*) **Conduction.** *'Conduction'* is the *transfer of heat from one part of a substance to another part of the same substance, or from one substance to another in physical contact with it, without appreciable displacement of molecules forming the substance.*

In *solids*, the heat is conducted by the following *two mechanisms :*

(*i*) ***By lattice vibration*** (The faster moving molecules or atoms in the hottest part of a body transfer heat by impacts some of their energy to adjacent molecules).

(*ii*) ***By transport of free electrons*** (Free electrons provide an energy flux in the direction of decreasing temperature—For metals, especially good electrical conductors, the electronic mechanism is responsible for the major portion of the heat flux except at low temperature).

In case of *gases*, the mechanisam of heat conduction is simple. The kinetic energy of a molecule is a function of temperature. These molecules are in a continuous random motion exchanging energy and momentum. When a molecule from the high temperature region collides with a molecule from the low temperature region, it loses energy by collisions.

In liquids, the mechanism of heat is nearer to that of gases. However, the molecules are more closely spaced and intermolecular forces come into play.

(*ii*) **Convection.** *'Convection' is the transfer of heat within a fluid by mixing of one portion of the fluid with another.*

- Convection is possible only in a fluid medium and is *directly linked* with the *transport of medium itself.*
- Convection constitutes the *macroform* of the heat transfer since macroscopic particles of a fluid moving in space cause the heat exchange.
- The effectiveness of heat transfer by convection depends largely upon the mixing motion of the fluid.

This mode of heat transfer is met with in situations where energy is transferred as heat to a flowing fluid at any surface over which flow occurs. This mode is *basically conduction in a very thin fluid layer at the surface and then mixing caused by the flow.* The heat flow depends on the properties of fluid and is independent of the properties of the material of the surface. However, the shape of the surface will influence the flow and hence the heat transfer.

Free or natural convection. *Free or natural convection occurs where the fluid circulates by virtue of the natural differences in densities of hot and cold fluids ; the denser portions of the fluid move downward because of the greater force of gravity, as compared with the force on the less dense.*

Forced convection. *When the work is done to blow or pump the fluid, it is said to be forced convection.*

(*iii*) **Radiation.** *'Radiation' is the transfer of heat through space or matter by means other than conduction or convection.*

Radiation heat is thought of as *electromagnetic waves or quanta* (as convenient) an emanation of the same nature as light and radio waves. *All bodies radiate heat ; so a transfer of heat by radiation occurs because hot body emits more heat than it receives and a cold body receives more heat than it emits.* Radiant *energy* (being electromagnetic radiation) *requires no medium for propagation and will pass through a vacuum.*

Note. The rapidly oscillating molecules of the hot body produce electromagnetic waves in hypothetical medium called *ether.* These waves are identical with light waves, radio waves and X-rays, differ from them only in *wavelength* and travel with an approximate velocity of 3×10^8 m/s. These waves carry energy with them and *transfer it to the relatively slow-moving molecules of the cold body* on which they happen to fall. The molecular energy of the later increases and results in a rise of its temperature. Heat travelling by radiation is known as *radiant heat.*

The properties of radiant heat in general, are similar to those of light. Some of the properties are :

(*i*) It does not require the presence of a material medium for its transmission.

(*ii*) Radiant heat can be reflected from the surfaces and obeys the ordinary laws of reflection.

(*iii*) It travels with velocity of light.

(*iv*) Like light, it shows interference, diffraction and polarisation etc.

(*v*) It follows the law of inverse square.

The wavelength of heat radiations is longer than that of light waves, hence they are invisible to the eye.

15.2. HEAT TRANSFER BY CONDUCTION

15.2.1. Fourier's Law of Heat Conduction

Fourier's law of heat conduction is an empirical law based on observation and states as follows :

"The rate of flow of heat through a simple homogeneous solid is directly proportional to the area of the section at right angles to the direction of heat flow, and to change of temperature with respect to the length of the path of the heat flow".

Mathematically, it can be represented by the equation :

$$Q \propto A \cdot \frac{dt}{dx}$$

where, Q = Heat flow through a body per unit time (in watts), W,

A = Surface area of heat flow (*perpendicular to the direction of flow*), m^2,

dt = Temperature difference of the faces of block (homogeneous solid) of thickness 'dx' through which heat flows,°C or K, and

dx = Thickness of body in the direction of flow, m.

Thus, $$Q = -k \cdot A \frac{dt}{dx} \quad ...(15.1)$$

where, k = Constant of proportionality and is known as *thermal conductivity of the body.*

The –ve sign of k [eqn. (15.1)] is to take care of the decreasing temperature alongwith the direction of increasing thickness or the direction of heat flow. The temperature gradient $\frac{dt}{dx}$ is *always negative along positive* x direction and therefore the value of Q *becomes* +ve.

Assumptions :

The following are the *assumptions* on which Fourier's law is based :

1. Conduction of heat takes place under *steady state conditions.*
2. The heat flow is unidirectional.
3. The temperatures gradient is *constant* and the temperature profile is *linear.*
4. There is no internal heat generation.
5. The bounding surfaces are isothermal in character.
6. The material is homogeneous and isotropic (*i.e.*, the value of thermal conductivity is *constant in all directions*).

Some essential features of Fourier's Law :

Following are some *essential features of Fourier's law* :

1. It is applicable to all matter (may be solid, liquid or gas).
2. It is based on experimental evidence and cannot be derived from first principle.
3. It is a vector expression indicating that heat flow rate is in the direction of decreasing temperature and is normal to an isotherm.
4. It helps to define thermal conductivity 'k' (transport property) of the medium through which heat is conducted.

15.2.2. Thermal Conductivity of Materials

From eqn. (15.1), we have $k = \frac{Q}{A} \cdot \frac{dx}{dt}$

The value of k = 1 when Q = 1, A = 1 and $\frac{dt}{dx} = 1$

Now $k = \frac{Q}{1} \cdot \frac{dx}{dt}$ (unit of k : W $\times \frac{1}{\text{m}^2} \times \frac{\text{m}}{\text{K(or °C)}}$ = W/mK. or W/m°C)

Thus, the *thermal conductivity of a material is defined* as follows :

"The amount of energy conducted through a body of unit area, and unit thickness in unit time when the difference in temperature between the faces causing heat flow is unit temperature difference".

It follows from eqn. (15.1) that materials with high thermal conductivities are good conductors of heat, whereas materials with low thermal conductives are good thermal insulator. *Conduction of heat occurs most readily in pure metals, less so in alloys, and much less readily in non-metals.* The very low thermal conductivities of certain thermal insulators *e.g.*, cork is due to their porosity, the air trapped within the material acting as an insulator.

Thermal conductivity (a property of material) depends essentially upon the following factors :

(*i*) Material structure (*ii*) Moisture content

(*iii*) Density of the material

(*iv*) Pressure and temperature (operating conditions)

Thermal conductivities (average values at normal pressure and temperature) of some common materials are as under :

Material	*Thermal conductivity (k) (W/mK)*	*Material*	*Thermal conductivity (k) (W/mK)*
1. Silver	410	8. Asbestos sheet	0.17
2. Copper	385	9. Ash	0.12
3. Aluminum	225	10. Cork, felt	0.05–0.10
4. Cast-iron	55–65	11. Saw dust	0.07
5. Steel	20–45	12. Glass wool	0.03
6. Concrete	1.20	13. Water	0.55–0.7
7. Glass (window)	0.75	14. Freon	0.0083

Following points *regarding thermal conductivity—its variation for different materials and under different conditions* are worth noting :

1. Thermal conductivity of a material is due to flow of free electrons (in case of *metals*) and lattice vibrational waves (in case of *fluids*).
2. Thermal conductivity in case of *pure metals* is the highest (k = 10 to 400 W/m°C). It *decreases with increase in impurity.*

   The range of k for other materials is as follows :

   Alloys : = k = 12 to 120 W/m°C

   Heat insulating and building materials : k = 0.023 to 2.9 W/m°C

   Liquids : k = 0.2 to 0.5 W/m°C

   Gases and vapours : k = 0.006 to 0.05 W/m°C.
3. Thermal conductivity of a metal varies considerably when it (metal) is heat treated or mechanically processed/formed.
4. Thermal conductivity of *most metals decreases with the increase in temperature* (*aluminium* and *uranium* being the *exceptions*).

   — In most of *liquids* the value of thermal conductivity tends to decrease with temperature (water being an exception) due to decrease in density with increase in temperature.

   — In case of gases the value of thermal conductivity *increases with temperature.* Gases with higher molecular weights have smaller thermal conductivities than with lower molecular weights. This is because the mean molecular path of gas molecules decreases with increase in density and k is directly proportional to the mean free path of the molecule.
5. The dependence of thermal conductivity (k) on temperature, for most materials is almost linear ;

$$k = k_0 (1 + \beta t) \qquad ...(15.2)$$

where, k_0 = Thermal conductivity at 0°C, and

β = Temperature coefficient of thermal conductivity, 1/°C (It is usually *positive for non-metals* and *insulating materials* (magnesite bricks being the exception) and *negative for metallic conductors* (aluminium and certain non-ferrous alloys are the exceptions).

6. In case of solids and liquids, thermal conductivity (k) is only very weakly dependent on pressure ; in case of gases the value of k is independent of pressure (near standard atmospheric).
7. In case of non-metallic solids :

   — Thermal conductivity of porous materials depends upon the type of gas or liquid present in the voids.

   — Thermal conductivity of a *damp material is considerably higher than that of the dry material and water taken individually.*

   — Thermal conductivity *increases with increase in density.*
8. The Wiedemann and Franz law (based on experiment results), regarding thermal and electrical conductivities of a material, states as follows :

   "The ratio of the thermal and electrical conductivities is the same for all metals at the same temperature ; and that the ratio is directly proportional to the absolute temperature of the metal."

Mathematically, $$\frac{k}{\sigma} \propto T$$

or $$\frac{k}{\sigma T} = C \quad \quad ...(15.3)$$

where, k = Thermal conductivity of metal at temperature T(K),

σ = Electrical conductivity of metal at temperature T(K), and

C = Constant (for all metals) is referred to as Lorenz number

(= 2.45×10^{-8} WΩ/K^2 ; Ω stands for ohms).

This law conveys that the materials which are *good conductors of electricity are also good conductors of heat.*

15.2.3. Thermal Resistance (R_{th})

When two physical systems are described by similar equations and have similar boundary conditions, these are said to be *analogous*. The heat transfer processes may be compared by *analogy* with the flow of electricity in an electrical resistance. As the flow of electric current in the electrical resistance is directly proportional to potential difference (dV) ; similarly heat flow rate, Q, is directly proportional to temperature difference (dt), the driving force for heat conduction through a medium.

As per Ohm's law (in electric-circuit theory), we have

$$\text{Current } (I) = \frac{\text{Potential difference } (dV)}{\text{Electrical resistance } (R)} \quad \quad ...(15.4)$$

By analogy, the heat flow equation (Fourier's equation) may be written as

$$\text{Heat flow rate } (Q) = \frac{\text{Temperature difference } (dt)}{\left(\dfrac{dx}{kA}\right)} \quad \quad ...(15.5)$$

By comparing eqns. (15.4) and (15.5), we find that I is analogus to, Q, dV is analogous to dt and R is analogous to the quantity $\left(\frac{dx}{kA}\right)$. The quantity $\frac{dx}{kA}$ is called *thermal conduction resistance* $(\mathbf{R_{th}})_{cond.}$ *i.e.*,

$$(R_{th})_{cond.} = \frac{dx}{kA}$$

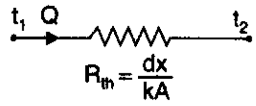


Fig. 15.1

The reciprocal of the thermal resistance is called *thermal conductance.*

It may be noted that *rules for combining electrical resistances in series and parallel apply equally well to thermal resistances.*

The concept of thermal resistance is quite helpful white making calculations for flow of heat.

15.2.4. General Heat Conduction Equation in Cartesian Coordinates

Consider an infinitesimal rectangular parallelopiped (volume element) of sides dx, dy and dz parallel, respectively, to the three axes (X, Y, Z) in a medium in which temperature is varying with location and time as shown in Fig. 15.2.

Let, t = Temperature at the left face $ABCD$; this temperature may be assumed uniform over the entire surface, since the area of this face can be made arbitrarily *small.*

$\frac{dt}{dx}$ = Temperature changes and rate of change along X-direction.

Then, $\left(\frac{\partial t}{\partial x}\right) dx$ = Change of temperature through distance dx, and

$t + \left(\frac{\partial t}{\partial x}\right) dx$ = Temperature on the right face $EFGH$ (at distance dx from the left face $ABCD$).

Further, let, k_x, k_y, k_z = Thermal conductivities (direction characteristics of the material) along X, Y and Z axes.

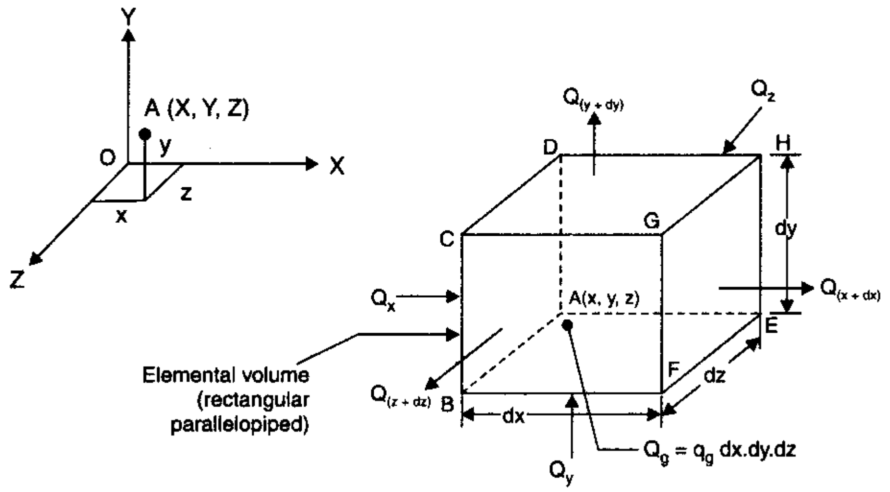


Fig. 15.2. Elemental volume for three-dimensional heat conduction analysis—Cartesian co-ordinates.

If the directional characteristics of a material are equal/same, it is called an *"Isotropic material"* and if unequal/different *'Anisotropic material'*.

q_g = Heat generated per unit volume per unit time.

Inside the control volume there may be heat sources due to flow of electric current in electric motors and generators, nuclear fission etc.

[**Note.** q_g may be function of position or time, or both].

ρ = Mass density of material.

c = Specific heat of the material.

Energy balance/equation for volume element :

Net heat accumulated in the element due to conduction of heat from all the coordinate directions considered (A) + heat generated within the element (B) = Energy stored in the element (C) ...(1)

Let, Q = Rate of *heat flow* in a direction, and

$Q' = (Q.d\tau)$ = *Total* heat flow (flux) in that direction (in time $d\tau$).

A. *Net heat accumulated in the element due to conduction of heat from all the directions considered :*

Quantity of heat flowing into the element from the left face $ABCD$ during the time interval $d\tau$ in X-direction is given by :

$$\text{Heat influx. } Q_x' = -k_x(dy.dz)\frac{\partial t}{\partial x}.d\tau \quad ...(i)$$

During the same time interval $d\tau$ the heat flowing out of the right face of control volume ($EFGH$) will be :

$$\text{Heat efflux. } Q'_{(x+dx)} = Q_x' + \frac{\partial}{\partial x}(Q_x')\,dx \quad ...(ii)$$

$\therefore$ Heat accumulation in the element due to heat flow in X-direction,

$$dQ_x' = Q_x' - \left[Q_x' + \frac{\partial}{\partial x}(Q_x')\,dx\right] \quad \text{[Subtracting (ii) from (i)]}$$

$$= -\frac{\partial}{\partial x}(Q_x')\,dx$$

$$= -\frac{\partial}{\partial x}\left[-k_x(dy.dz)\frac{\partial t}{\partial x}.d\tau\right]dx$$

$$= \frac{\partial}{\partial x}\left[k_x\frac{\partial t}{\partial x}\right]dx.dy.dz.d\tau \quad ...(15.6)$$

Similarly the heat accumulated due to heat flow by conduction along Y and Z directions in time $d\tau$ will be :

$$dQ_y' = \frac{\partial}{\partial x}\left[k_y\frac{\partial t}{\partial y}\right]dx.dy.dz.d\tau \quad ...(15.7)$$

$$dQ_z' = \frac{\partial}{\partial z}\left[k_z\frac{\partial t}{\partial z}\right]dx.dy.dz.d\tau \quad ...(15.8)$$

$\therefore$ *Net heat accumulated* in the element due to conduction of heat from all the co-ordinate directions considered

$$= \frac{\partial}{\partial x}\left[k_x \frac{\partial t}{\partial x}\right] dx.dy.dz.d\tau + \frac{\partial}{\partial y}\left[k_y \frac{\partial t}{\partial y}\right] dx.dy.dz.d\tau + \frac{\partial}{\partial z}\left[k_z \frac{\partial t}{\partial z}\right] dx.dy.dz.d\tau$$

$$= \left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right)\right] dx.dy.dz.d\tau \quad ...(15.9)$$

B. *Total heat generated within the element* (Q_g') :

The total heat generated in the element is given by :

$$Q_g' = q_g(dx.dy.dz)\, d\tau \quad ...(15.10)$$

C. *Energy stored in the element :*

The total heat accumulated in the element due to heat flow along coordinate axes (eqn. 15.9) and the haet generated within the element (eqn. 15.10) together serve to increase the thermal energy of the element/lattice. This increase in thermal energy is given by :

$$\rho(dx.dy.dz)c.\frac{\partial t}{\partial \tau}.d\tau \quad ...(15.11)$$

[$\because$ Heat stored in the body = Mass of the body × specific heat of the body material × rise in the temperature of body].

Now, substituting eqns. (15.9), (15.10), (15.11), in the eqn. (1), we have

$$\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right)\right] dx.dy.dz.d\tau + q_g(dx.dy.dz)d\tau = \rho(dx.dy.dz)c.\frac{\partial t}{\partial \tau}.d\tau$$

Dividing both sides by $dx.dy.dz.d\tau$, we have

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right) + q_g = \rho.c.\frac{\partial t}{\partial \tau} \quad ...(15.12)$$

or, using the vector operator ∇, we get

$$\nabla . (k\nabla t) + q_g = \rho.c.\frac{\partial t}{\partial \tau}$$

This is known as the **general heat conduction equation** for **'non-homogeneous material', self heat generating'** and **'unsteady three-dimensional flow'**. This equation establishes in *differential form the relationship between the time and space variation of temperature at any point of solid through which heat flow by conduction takes place.*

General heat conduction equation for constant thermal conductivity :

In case of *homogeneous* (in which properties *e.g.*, specific heat, density, thermal conductivity etc. are same everywhere in the material) and *isotropic* (in which properties are independent of surface orientation) material, $k_x = k_y = k_z = k$ and diffusion equation eqn. (15.12) becomes

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{\rho.c}{k}.\frac{\partial t}{\partial \tau} = \frac{1}{\alpha}.\frac{\partial t}{\partial \tau} \quad ...(15.13)$$

where $\alpha = \dfrac{k}{\rho.c} = \dfrac{\text{Thermal conductivity}}{\text{Thermal capacity}}$

The quantity, $\alpha = \dfrac{k}{\rho.c}$ is known as **thermal diffusivity.**

— The larger the value of α, the faster will the heat diffuse through the material and its temperature will change with time. This will result either due to a high value of thermal conductivity k or a low value of heat capacity $\rho.c$. A low value of heat capacity means the less amount of heat entering the element would be absorbed and used to raise its temperature and more would be available for onward transmission. Metals and gases have relatively high value of α and their response to temperature changes is quite rapid. The non-metallic solids and liquids respond slowly to temperature changes because of their relatively small value of thermal diffusivity.

— Thermal diffusivity is an important characteristic quantity for *unsteady conduction situations.*

Eqn. (15.13) by using Laplacian ∇^2, may be written as :

$$\nabla^2 t + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{...[15.13 (a)]}$$

Eqn. (15.13), governs the temperature distribution under unsteady heat flow through a material which is homogeneous and isotropic.

Other simplified forms of heat conduction equation in cartesian co-ordinates :

(*i*) For the case when *no internal source of heat generation is present.* Eqn. (15.13) reduces to $\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$ [Unsteady state $\left(\frac{\partial t}{\partial \tau} \neq 0\right)$ heat flow with no internal heat generation]

or

$$\nabla^2 t = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{...(Fourier's equation)} \quad \text{...(15.14)}$$

(*ii*) Under the situations when temperature does not depend on time, the conduction then takes place in steady state $\left(i.e., \frac{\partial t}{\partial \tau} = 0\right)$ and the eqn. (15.13) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = 0$$

or

$$\nabla^2 t + \frac{q_g}{k} = 0 \quad \text{...(Poisson's equation)} \quad \text{...(15.15)}$$

In the absence of internal heat generation, eqn. (15.15) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = 0$$

or

$$\nabla^2 t = 0 \quad \text{...(Laplace equation)} \quad \text{...(15.16)}$$

(*iii*) *Steady state and one-dimensional heat transfer*

$$\frac{\partial^2 t}{\partial x^2} + \frac{q_g}{k} = 0 \quad \text{...(15.17)}$$

(*iv*) *Steady state, one-dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} = 0 \quad \text{...(15.18)}$$

(*v*) *Steady state, two dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0 \quad \text{...(15.19)}$$

(*vi*) *Unsteady state, one dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{...(15.20)}$$

15.2.5. Heat Conduction Through Plane and Composite Walls

15.2.5.1. Heat conduction through a plane wall

Refer Fig. 15.3 (*a*). Consider a plane wall of homogeneous material through which heat is flowing *only in x-direction.*

Let, L = Thickness of the plane wall,

A = Cross-sectional area of the wall,

k = Thermal conductivity of the wall material, and

t_1, t_2 = Temperatures maintained at the two faces 1 and 2 of the wall, respectively.

The general heat conduction equation in cartesian coordinates is given by :

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{...[Eqn. 15.13]}$$

If the heat conduction takes place under the conditions,

steady state $\left(\frac{\partial t}{\partial \tau} = 0\right)$, one-dimensional $\left[\frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t}{\partial z^2} = 0\right]$ and with no internal heat generation $\left(\frac{q_g}{k} = 0\right)$ then the above equation is reduced to :

$$\frac{\partial^2 t}{\partial x^2} = 0, \quad \text{or} \quad \frac{d^2 t}{dx^2} = 0 \quad \text{...(15.21)}$$

By integrating the above differential twice, we have

$$\frac{\partial t}{\partial x} = C_1 \text{ and } t = C_1 x + C_2 \quad \text{...(15.22)}$$

where C_1 and C_2 are the arbitrary constants. The values of these constants may be calculated from the known boundary conditions as follows :

At $x = 0$ $\quad t = t_1$

At $x = L$ $\quad t = t_2$

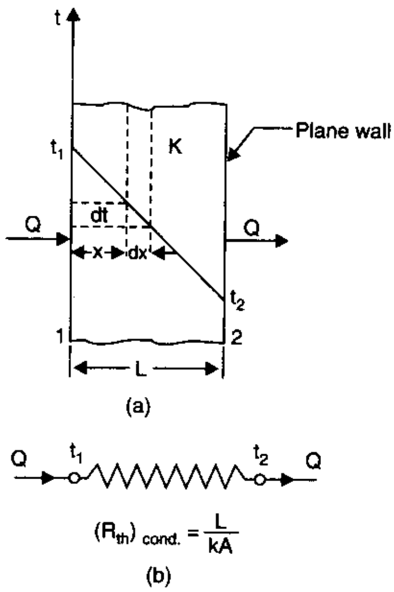


Fig. 15.3. Heat conduction through a plane wall.

Substituting the values in the eqn. (15.22), we get

$$t_1 = O + C_2 \text{ and } t_2 = C_1 L + C_2$$

After simplification, we have, $C_2 = t_1$ and $C_1 = \frac{t_2 - t_1}{L}$

Thus, the eqn. (15.22) reduces to :

$$t = \left(\frac{t_2 - t_1}{L}\right)x + t_1 \quad \text{...(15.23)}$$

The eqn. (15.23) indicates that *temperature distribution across a wall is linear* and is *independent of thermal conductivity.* Now heat through the plane wall can be found by using Fourier's equation as follows :

$$Q = -kA \frac{dt}{dx}, \left(\text{where } \frac{dt}{dx} = \text{temperature gradient}\right) \quad \text{...[Eqn. (1.1)]}$$

But, $$\frac{dt}{dx} = \frac{d}{dx}\left[\left(\frac{t_2 - t_1}{L}\right)x + t_1\right] = \frac{t_2 - t_1}{L}$$

$\therefore$ $$Q = -kA\frac{(t_2 - t_1)}{L} = \frac{kA(t_1 - t_2)}{L} \quad ...(15.24)$$

Eqn. (15.24) can be written as :

$$Q = \frac{(t_1 - t_2)}{(L/kA)} = \frac{(t_1 - t_2)}{(R_{th})_{cond.}} \quad ...(15.25)$$

where, $(R_{th})_{cond.}$ = Thermal resistance to heat conduction. Fig. 15.3 (*b*) shows the *equivalent thermal circuit* for heat flow through the plane wall.

Let us now find out the condition when instead of space, weight is the main criterion for selection of the insulation of a plane wall.

Thermal resistance (conduction) of the wall, $(R_{th})_{cond.} = \frac{L}{kA}$ ...(*i*)

Weight of the wall, $W = \rho A L$ ...(*ii*)

Eliminating L from (*i*) and (*ii*), we get

$$W = \rho A.(R_{th})_{cond.}\ kA = (\rho.k)A^2.(R_{th})_{cond.} \quad ...(15.26)$$

The eqn. (15.26) stipulates the condition that, for a specified thermal resistance, the *lightest insulation will be one which has the smallest product of density* (ρ) *and thermal conductivity* (k).

15.2.5.2. Heat conduction through a composite wall

Refer Fig. 15.4 (*a*). Consider the transmission of heat through a composite wall consisting of a number of slabs.

Let L_A, L_B, L_C = Thicknesses of slabs A, B and C respectively (also called path lengths),

k_A, k_B, k_C = Thermal conductivities of the slabs A, B and C respectively,

t_1, $t_4 (t_1 > t_4)$ = Temperatures at the wall surfaces 1 and 4 respectively, and

t_2, t_3 = Temperatures at the interfaces 2 and 3 respectively.

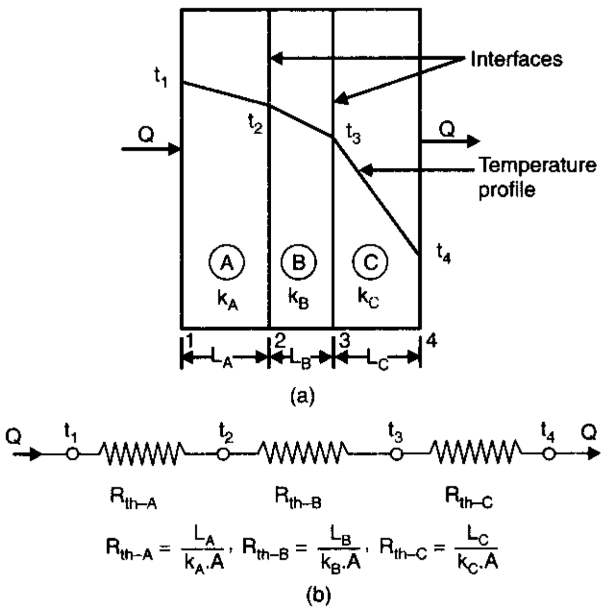


Fig. 15.4. Steady state conduction through a composite wall.

Since the quantity of heat transmitted per unit time through each slab/layer is same, we have

$$Q = \frac{k_A . A\,(t_1 - t_2)}{L_A} = \frac{k_B . A\,(t_2 - t_3)}{L_B} = \frac{k_C . A\,(t_3 - t_4)}{L_C}$$

(Assuming that there is a perfect contact between the layers and no temperature drop occurs across the interface between the materials).

Rearranging the above expression, we get

$$t_1 - t_2 = \frac{Q . L_A}{k_A . A} \quad ...(i)$$

$$t_2 - t_3 = \frac{Q . L_B}{k_B . A} \quad ...(ii)$$

$$t_3 - t_4 = \frac{Q . L_C}{k_C . A} \quad ...(iii)$$

Adding (*i*), (*ii*) and (*iii*), we have

$$(t_1 - t_4) = Q\left[\frac{L_A}{k_A . A} + \frac{L_B}{k_B . A} + \frac{L_C}{k_C . A}\right]$$

or

$$Q = \frac{A\,(t_1 - t_4)}{\left[\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}\right]} \quad ...(15.27)$$

or

$$Q = \frac{(t_1 - t_4)}{\left[\frac{L_A}{k_A . A} + \frac{L_B}{k_B . A} + \frac{L_C}{k_C . A}\right]} = \frac{(t_1 - t_4)}{[R_{th-A} + R_{th-B} + R_{th-C}]} \quad ...(15.28)$$

If the composite wall consists of n slabs/layers, then

$$Q = \frac{[t_1 - t_{(n+1)}]}{\sum_1^n \frac{L}{kA}} \quad ...(15.29)$$

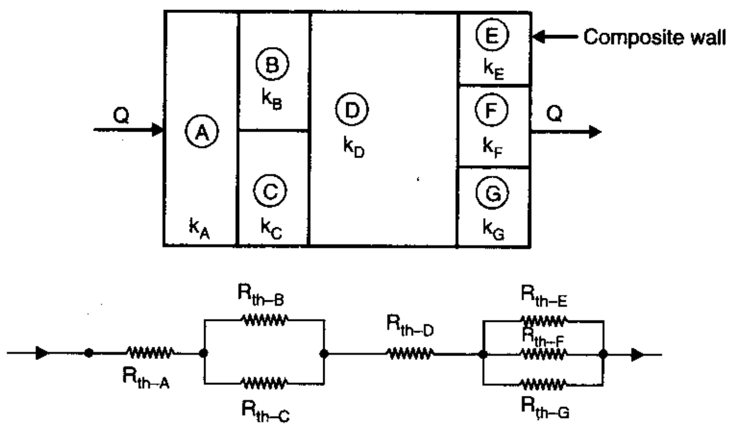


Fig. 15.5. Series and parallel one-dimensional heat transfer through a composite wall and electrical analog.

In order to solve more complex problems involving both series and parallel thermal resistances, the electrical analogy may be used. A typical problem and its analogous electric circuit are shown in Fig. 15.5.

$$Q = \frac{\Delta t_{\text{overall}}}{\Sigma R_{th}} \qquad \text{...(15.30)}$$

Thermal contact resistance. In a composite (multi-layer) wall, the calculations of heat flow are made on the assumptions : (*i*) The contact between the adjacent layers is perfect, (*ii*) At the interface there is no fall of temperature, and (*iii*) At the interface the temperature is continuous, although there is discontinuity in temperature gradient. In real systems, however, due to surface roughness and void spaces (usually filled with air) the contact surfaces *touch only at discrete locations.* Thus there is not a single plane of contact, which means that the area available for the flow of heat at the interface will be small compared to geometric face area. Due to this reduced area and presence of air voids, *a large resistance to heat flow at the interface occurs.* This resistance is known as *thermal contact resistance* and it *causes temperature drop between two materials at the interface* as shown in Fig. 15.6.

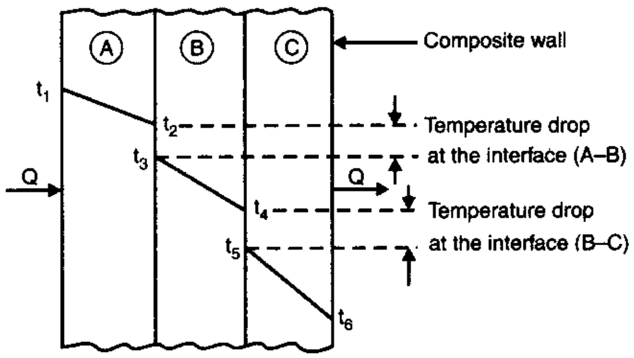


Fig. 15.6. Temperature drops at the interfaces.

Refer Fig. 15.6. The contact resistances are given by

$$(R_{th\text{-}AB})_{\text{cond.}} = \frac{(t_2 - t_3)}{Q/A} \text{ and } (R_{th\text{-}BC})_{\text{cont.}} = \frac{(t_4 - t_5)}{Q/A}.$$

15.2.6. The Overall Heat-transfer Coefficient

While dealing with the problems of fluid to fluid heat transfer across a metal boundary, it is usual to adopt an overall heat transfer coefficient U which *gives the heat transmitted per unit area per unit time per degree temperature difference between the bulk fluids on each side of the metal.*

Refer Fig. 15.7.

Let, L = Thickness of the metal wall,

k = Thermal conductivity of the wall material,

t_1 = Temperature of the surface-1,

t_2 = Temperature of the surface-2,

t_{hf} = Temperature of the hot fluid,

t_{cf} = Temperature of the cold fluid,

h_{hf} = Heat transfer coefficient from hot fluid to metal surface, and

h_{cf} = Heat transfer coefficient from metal surface to cold fluid.

(The suffices *hf* and *cf* stand for hot fluid and cold fluid respectivley.)

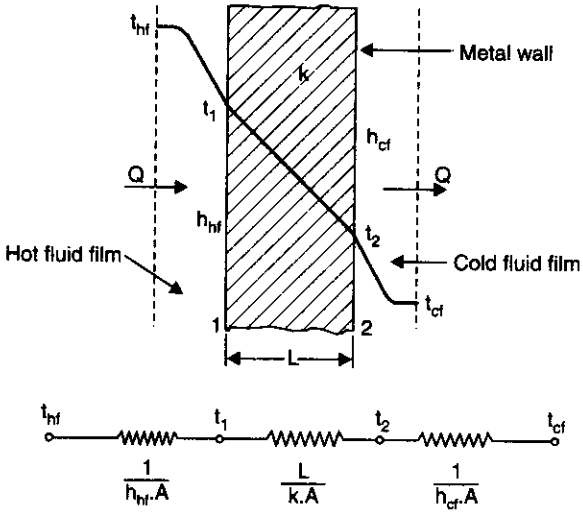


Fig. 15.7. The overall heat transfer through a plane wall.

The equations of heat flow through the fluid and the metal surface are given by

$$Q = h_{hf} \cdot A(t_{hf} - t_1) \quad \text{...}(i)$$

$$Q = \frac{k_A (t_1 - t_2)}{L} \quad \text{...}(ii)$$

$$Q = h_{cf} \cdot A(t_2 - t_{cf}) \quad \text{...}(iii)$$

By rearranging (*i*), (*ii*) and (*iii*), we get

$$t_{hf} - t_1 = \frac{Q}{h_{hf}.A} \quad \text{...}(iv)$$

$$t_1 - t_2 = \frac{QL}{k \,.\, A} \quad \text{...}(v)$$

$$t_2 - t_{cf} = \frac{Q}{h_{cf} \,.\, A} \quad \text{...}(vi)$$

Adding (*iv*), (*v*) and (*vi*), we get

$$t_{hf} - t_{cf} = Q \left[\frac{1}{h_{hf} \,.\, A} + \frac{L}{k \,.\, A} + \frac{1}{h_{cf} \,.\, A} \right]$$

or

$$Q = \frac{A\,(t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}} \quad \text{...(15.31)}$$

If U is the overall coefficient of heat transfer, then

$$Q = U.A\,(t_{hf} - t_{cf}) = \frac{A\,(t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

or

$$U = \frac{1}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}} \quad \text{...(15.32)}$$

It may be noticed from the above equation that if the individual coefficients differ greatly in magnitude only a change in the *least* will have significant effect on the rate of heat transfer.

Example 15.1. *The inner surface of a plane brick wall is at 60°C and the outer surface is at 35°C. Calculate the rate of heat transfer per m^2 of surface area of the wall, which is 220 mm thick. The thermal conductivity of the brick is 0.51 W/m°C.*

Solution. Temperature of the inner surface of the wall t_1 = 60°C.

Temperature of the outer surface of the wall,

$$t_2 = 35°C$$

The thickness of the wall, L = 220 mm = 0.22 m

Thermal conductivity of the brick,

$$k = 0.51 \text{ W/m°C}$$

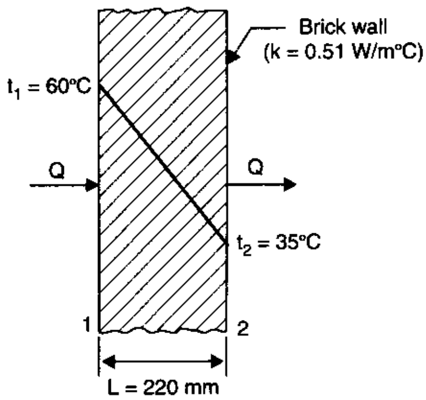


Fig. 15.8

Rate of heat transfer per m², q :

Rate of heat transfer per unit area,

$$q = \frac{Q}{A} = \frac{k(t_1 - t_2)}{L}$$

or $$q = \frac{0.51 \times (60 - 35)}{0.22} = \mathbf{57.95\ W/m^2}. \quad \textbf{(Ans.)}$$

Example 15.2. *A reactor's wall 320 mm thick, is made up of an inner layer of fire brick (k = 0.84 W/m°C) covered with a layer of insulation (k = 0.16 W/m°C). The reactor operates at a temperature of 1325°C and the ambient temperature is 25°C.*

(i) Determine the thickness of fire brick and insulation which gives minimum heat loss.

(ii) Calculate the heat loss presuming that the insulating material has a maximum temperature of 1200°C.

If the calculated heat loss is not acceptable, then state whether addition of another layer of insulation would provide a satisfactory solution.

Solution. Refer Fig. 15.9.

Given : t_1 = 1325°C ; t_2 = 1200°C, t_3 = 25°C ;

$L_A + L_B = L$ = 320 mm or 0.32 m

$\therefore \quad L_B = (0.32 - L_A)$; ...(*i*)

k_A = 0.84 W/m°C ;

k_B = 0.16 W/m°C.

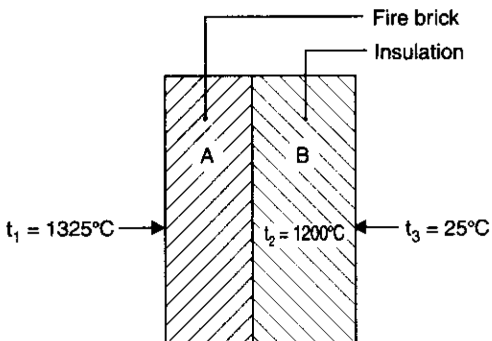


Fig. 15.9

(*i*) **L_A : ; L_B :**

The heat flux, under steady state conditions, is constant throughout the wall and is same for each layer. Then for *unit area* of wall,

$$q = \frac{t_1 - t_3}{L_A/k_A + L_B/k_B} = \frac{t_1 - t_2}{L_A/k_A} = \frac{t_2 - t_3}{L_B/k_B}$$

Considering first two quantities, we have

$$\frac{(1325 - 25)}{L_A/0.84 + L_B/0.16} = \frac{(1325 - 1200)}{L_A/0.84}$$

or $$\frac{1300}{1.190\,L_A + 6.25(0.32 - L_A)} = \frac{105}{L_A}$$

or
$$\frac{1300}{1.190\,L_A + 2 - 6.25\,L_A} = \frac{105}{L_A}$$

or
$$\frac{1300}{2 - 5.06\,L_A} = \frac{105}{L_A}$$

or
$$1300 L_A = 105\,(2 - 5.06\,L_A)$$

or
$$1300\,L_A = 210 - 531.3\,L_A$$

or
$$L_A = \frac{210}{(1300 + 531.3)} = 0.1146 \text{ m or } \mathbf{114.6 \text{ mm.}} \textbf{ (Ans.)}$$

∴ Thickness of insulation $L_B = 320 - 114.6 =$ **205.4 mm. (Ans.)**

(*ii*) **Heat loss per unit area,** q :

Heat loss per unit area, $q = \dfrac{t_1 - t_2}{L_A/k_A} = \dfrac{1325 - 1200}{0.1146/0.84} =$ **916.23 W/m². (Ans.)**

If another layer of insulating material is added, the heat loss from the wall will reduce ; consequently the temperature drop across the fire brick lining will drop and the interface temperature t_2 will *rise*. As the interface temperature is *already fixed*. Therefore, a *satisfactory solution will not be available* by *adding layer of insulation.*

Example 15.3. *An exterior wall of a house may be approximated by a 0.1 m layer of common brick (k = 0.7 W/m°C) followed by a 0.04 m layer of gypsum plaster (k = 0.48 W/m°C). What thickness of loosely packed rock wool insulation (k = 0.065 W/m°C) should be added to reduce the heat loss or (gain) through the wall by 80 per cent ?* **(AMIE Summer, 1997)**

Solution. Refer Fig. 15.10.

Thickness of common brick, $L_A = 0.1$ m

Thickness of gypsum plaster, $L_B = 0.04$ m

Thickness of rock wool, $L_C = x$ (in m) = ?

Thermal conductivities :

Common brick, $k_A = 0.7$ W/m°C

Gypsum plaster, $k_B = 0.48$ W/m°C

Rock wool, $k_C = 0.065$ W/m°C

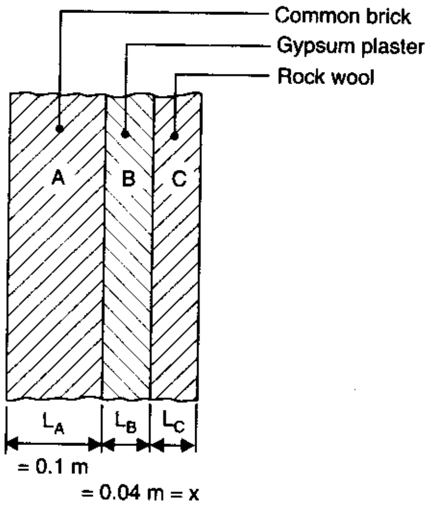


Fig. 15.10

Case I. *Rock wool insulation* **not** *used :*

$$Q_1 = \frac{A\,(\Delta t)}{\dfrac{L_A}{k_A} + \dfrac{L_B}{k_B}} = \frac{A(\Delta t)}{\dfrac{0.1}{0.7} + \dfrac{0.04}{0.48}} \quad ...(i)$$

Case II. *Rock wool insulation used :*

$$Q_2 = \frac{A\,(\Delta t)}{\dfrac{L_A}{k_A} + \dfrac{L_B}{k_B} + \dfrac{L_C}{k_C}} = \frac{A(\Delta t)}{\dfrac{0.1}{0.7} + \dfrac{0.04}{0.48} + \dfrac{x}{0.065}} \quad ...(ii)$$

But $Q_2 = (1 - 0.8)Q_1 = 0.2\,Q_1$ ...(Given)

∴
$$\frac{A(\Delta t)}{\dfrac{0.1}{0.7} + \dfrac{0.04}{0.48} + \dfrac{x}{0.065}} = 0.2 \times \frac{A(\Delta t)}{\dfrac{0.1}{0.7} + \dfrac{0.04}{0.48}}$$

or
$$\frac{0.1}{0.7} + \frac{0.04}{0.48} = 0.2\left[\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}\right]$$

or $$0.1428 + 0.0833 = 0.2\ [0.1428 + 0.0833 + 15.385\ x]$$

or $$0.2261 = 0.2\ (0.2261 + 15.385\ x)$$

or $$x = 0.0588 \text{ m or } 58.8 \text{ mm}$$

Thus, the *thickness of rock wool insulation should be* **58.8 mm. (Ans.)**

Example 15.4. *A furnace wall consists of 200 mm layer of refractory bricks, 6 mm layer of steel plate and a 100 mm layer of insulation bricks. The maximum temperature of the wall is 1150°C on the furnace side and the minimum temperature is 40°C on the outermost side of the wall. An accurate energy balance over the furnace shows that the heat loss from the wall is 400 W/m². It is known that there is a thin layer of air between the layers of refractory bricks and steel plate. Thermal conductivities for the three layers are 1.52, 45 and 0.138 W/m°C respectively. Find :*

(i) To how many millimetres of insulation brick is the air layer equivalent ?

(ii) What is the temperature of the outer surface of the steel plate ?

Solution. Refer Fig. 15.11.

Thickness of refractory bricks, $L_A = 200 \text{ mm} = 0.2 \text{ m}$

Thickness of steel plate, $L_C = 6 \text{ mm} = 0.006 \text{ m}$

Thickness of insulation bricks, $L_D = 100 \text{ mm} = 0.1 \text{ m}$

Difference of temperature between the innermost and outermost side of the wall,

$$\Delta t = 1150 - 40 = 1110°\text{C}$$

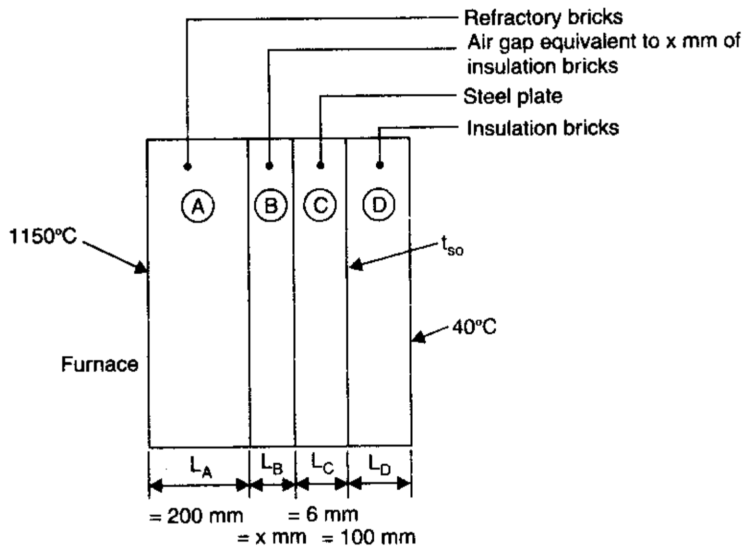


Fig. 15.11

Thermal conductivities :

$$k_A = 1.52 \text{ W/m°C} ;\ k_B = k_D = 0.138 \text{ W/m°C} ;\ k_C = 45 \text{ W/m°C}$$

Heat loss from the wall, $q = 400 \text{ W/m}^2$

(*i*) **The value of x = (L_C) :**

We know, $$Q = \frac{A \cdot \Delta t}{\Sigma \frac{L}{k}} \text{ or } \frac{Q}{A} = q = \frac{\Delta t}{\Sigma \frac{L}{k}}$$

or
$$400 = \frac{1110}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{L_D}{k_D}}$$

or
$$400 = \frac{1110}{\frac{0.2}{1.52} + \frac{(x/1000)}{0.138} + \frac{0.006}{45} + \frac{0.1}{0.138}}$$
$$= \frac{1110}{0.1316 + 0.0072x + 0.00013 + 0.7246} = \frac{1110}{0.8563 + 0.0072x}$$

or
$$0.8563 + 0.0072\,x = \frac{1110}{400} = 2.775$$

or
$$x = \frac{2.775 - 0.8563}{0.0072} = \mathbf{266.5\ mm.}\quad \textbf{(Ans.)}$$

(*ii*) **Temperature of the outer surface of the steel plate t_{so} :**

$$q = 400 = \frac{(t_{so} - 40)}{L_D/k_D}$$

or
$$400 = \frac{(t_{so} - 40)}{(0.1/0.138)} = 1.38(t_{so} - 40)$$

or
$$t_{so} = \frac{400}{1.38} + 40 = \mathbf{329.8°C.}\quad \textbf{(Ans.)}$$

Example 15.5. *Find the heat flow rate through the composite wall as shown in Fig. 15.12. Assume one dimensional flow.*

k_A = *150 W/m°C,*
k_B = *30 W/m°C,*
k_C = *65 W/m°C and*
k_D = *50 W/m°C.*

(M.U. Winter, 1997)

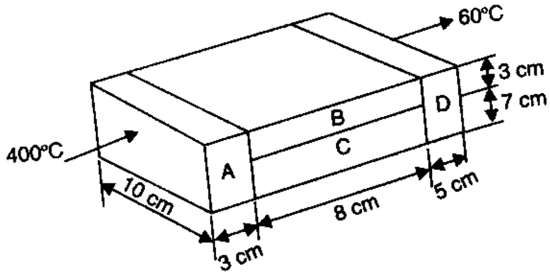


Fig. 15.12

Solution. The thermal circuit for heat flow in the given composite system (shown in Fig. 15.12) has been illustrated in Fig. 15.13.

Thickness :

L_A = 3 cm = 0.03 m ; $L_B = L_C$ = 8 cm = 0.08 m ; L_D = 5 cm = 0.05 m

Areas :

$A_A = 0.1 \times 0.1 = 0.01\ m^2$; $A_B = 0.1 \times 0.03 = 0.003\ m^2$
$A_C = 0.1 \times 0.07 = 0.007\ m^2$; $A_D = 0.1 \times 0.1 = 0.01\ m^2$

Heat flow rate, Q :

The thermal resistances are given by

$$R_{th\text{-}A} = \frac{L_A}{k_A A_A} = \frac{0.03}{150 \times 0.01} = 0.02$$

$$R_{th\text{-}B} = \frac{L_B}{k_B A_B} = \frac{0.08}{30 \times 0.003} = 0.89$$

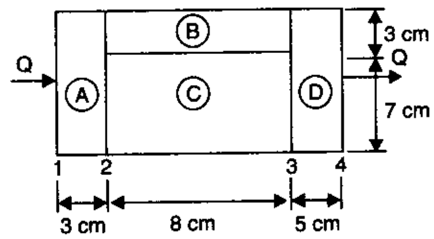
$$R_{th-C} = \frac{L_C}{k_C A_C} = \frac{0.08}{65 \times 0.007} = 0.176$$

$$R_{th-D} = \frac{L_D}{k_D A_D} = \frac{0.05}{50 \times 0.01} = 0.1$$

The equivalent thermal resistance for the parallel thermal resistances R_{th-B} and R_{th-C} is given by :

$$\frac{1}{(R_{th})_{eq.}} = \frac{1}{R_{th-B}} + \frac{1}{R_{th-C}} = \frac{1}{0.89} + \frac{1}{0.176}$$
$$= 6.805$$

$$\therefore \quad (R_{th})_{eq.} = \frac{1}{6.805} = 0.147$$



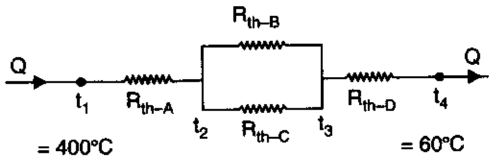


Fig. 15.13. Thermal circuit for heat flow in the composite system.

Now, the total thermal resistance is given by

$$(R_{th})_{total} = R_{th-A} + (R_{th})_{eq.} + R_{th-D}$$
$$= 0.02 + 0.147 + 0.1 = 0.267$$

$$\therefore \quad Q = \frac{(\Delta t)_{overall}}{(R_{th})_{total}} = \frac{(400-60)}{0.267} = \mathbf{1273.4\ W.} \quad \textbf{(Ans.)}$$

Example 15.6. *A mild steel tank of wall thickness 12 mm contains water at 95°C. The thermal conductivity of mild steel is 50 W/m°C, and the heat transfer coefficients for the inside and outside the tank are 2850 and 10 W/m²°C, respectively. If the atmospheric temperature is 15°C, calculate :*

(i) The rate of heat loss per m² of the tank surface area ;

(ii) The temperature of the outside surface of the tank.

Solution. Refer Fig. 15.14.

Thickness of mild steel tank wall

$$L = 12 \text{ mm} = 0.012 \text{ m}$$

Temperature of water, $t_{hf} = 95°C$

Temperature of air, $t_{cf} = 15°C$

Thermal conductivity of mild steel,

$$k = 50 \text{ W/m°C}$$

Heat transfer coefficients :

Hot fluid (water), $h_{hf} = 2850$ W/m²°C

Cold fluid (air), $h_{cf} = 10$ W/m²°C

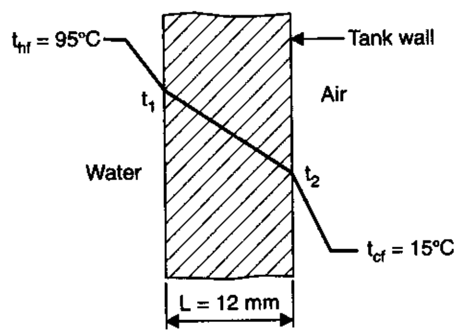


Fig. 15.14

(*i*) **Rate of heat loss per m² of the tank surface area, q :**

Rate of heat loss per m² of tank surface,

$$q = UA(t_{hf} - t_{cf})$$

The overall heat transfer coefficient, U is found from the relation ;

$$\frac{1}{U} = \frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}} = \frac{1}{2850} + \frac{0.012}{50} + \frac{1}{10}$$
$$= 0.0003508 + 0.00024 + 0.1 = 0.1006$$

$\therefore \quad U = \dfrac{1}{0.1006} = 9.94 \text{ W/m}^2\text{°C}$

$\therefore \quad q = 9.94 \times 1 \times (95 - 15) = $ **795.2 W/m².** **(Ans.)**

(*ii*) **Temperature of the outside surface of the tank, t_2 :**

We know that, $\quad q = h_{cf} \times 1 \times (t_2 - t_{cf})$

or $\quad 795.2 = 10(t_2 - 15)$

or $\quad t_2 = \dfrac{795.2}{10} + 15 = $ **94.52°C.** **(Ans.)**

Example 15.7. *The interior of a refrigerator having inside dimensions of 0.5 m × 0.5 m base area and 1 m height, is to be maintained at 6°C. The walls of the refrigerator are constructed of two mild steel sheets 3 mm thick (k = 46.5 W/m°C) with 50 mm of glass wool insulation (k = 0.046 W/m°C) between them. If the average heat transfer coefficients at the inner and outer surfaces are 11.6 W/m²°C and 14.5 W/m²°C respectively, calculate :*

(*i*) *The rate at which heat must be removed from the interior to maintain the specified temperature in the kitchen at 25°C, and*

(*ii*) *The temperature on the outer surface of the metal sheet.*

Solution. Refer Fig. 15.15

Given : $\quad L_A = L_C = 3 \text{ mm} = 0.003 \text{ m}$;

$L_B = 50 \text{ mm} = 0.05 \text{ m}$;

$k_A = k_C = 46.5 \text{ W/m°C}$; $k_B = 0.046 \text{ W/m°C}$;

$h_0 = 11.6 \text{ W/m}^2\text{°C}$; $h_i = 14.5 \text{ W/m}^2\text{°C}$;

$t_0 = 25\text{°C}$; $t_i = 6\text{°C}$.

The total area through which heat is coming into the refrigerator

$$A = 0.5 \times 0.5 \times 2 + 0.5 \times 1 \times 4 = 2.5 \text{ m}^2$$

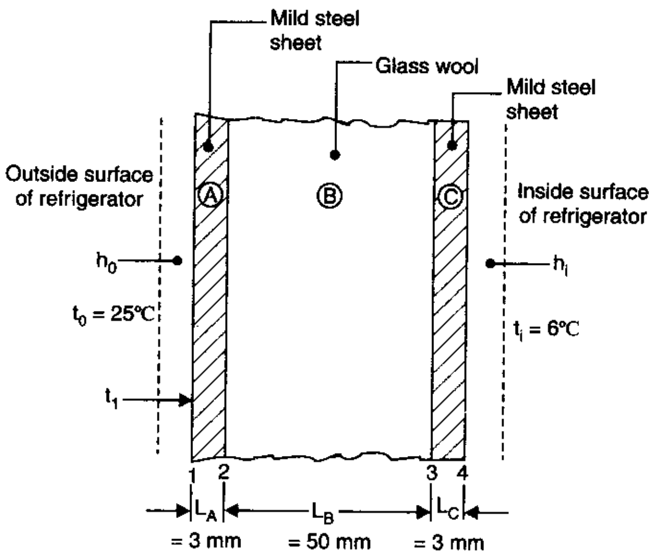


Fig. 15.15

(*i*) **The rate of removal of heat, Q :**

$$Q = \frac{A(t_0 - t_i)}{\frac{1}{h_o} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_i}}$$

$$= \frac{2.5\,(25-6)}{\frac{1}{11.6} + \frac{0.003}{46.5} + \frac{0.05}{0.046} + \frac{0.003}{46.5} + \frac{1}{14.5}} = \mathbf{38.2\ W.}\quad \textbf{(Ans.)}$$

(*ii*) **The temperature at the outer surface of the metal sheet, t_1 :**

$$Q = h_o A(25 - t_1)$$

or $$38.2 = 11.6 \times 2.5\,(25 - t_1)$$

or $$t_1 = 25 - \frac{38.2}{11.6 \times 2.5} = \mathbf{23.68°C.}\quad \textbf{(Ans.)}$$

Example 15.8. *A furnace wall is made up of three layers of thicknesses 250 mm, 100 mm and 150 mm with thermal conductivities of 1.65, k and 9.2 W/m°C respectively. The inside is exposed to gases at 1250°C with a convection coefficient of 25 W/m²°C and the inside surface is at 1100°C, the outside surface is exposed air at 25°C with convection coefficient of 12 W/m²°C. Determine :*

(*i*) *The unknown thermal conductivity 'k' ;*

(*ii*) *The overall heat transfer coefficient ;*

(*iii*) *All surface temperatures.*

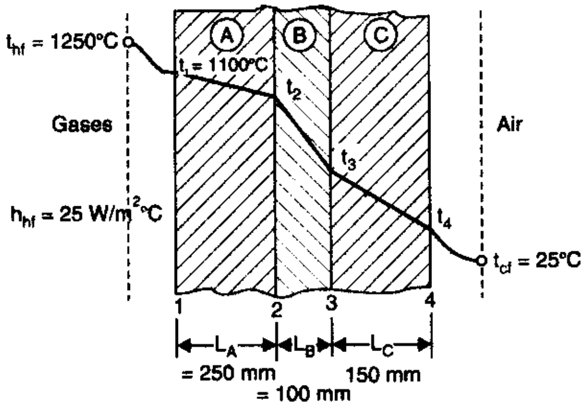
Solution. L_A = 250 mm 0.25 m ; L_B = 100 mm = 0.1 m ;

L_C = 150 mm = 0.15 m ; k_A = 1.65 W/m°C ;

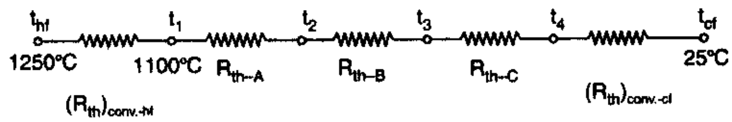
k_C = 9.2 W/m°C ; t_{hf} = 1250°C ; t_1 = 1100°C

h_{hf} = 25 W/m²°C ; h_{cf} = 12 W/m²°C

(*i*) **Thermal conductivity, k (= k_B) :**



(*a*) Composite system.



(*b*) Thermal circuit.

Fig. 15.16

The rate of heat transfer per unit area of the furnace wall,

$$q = h_{hf}(t_{hf} - t_1)$$
$$= 25(1250 - 1100) = 3750 \text{ W/m}^2$$

Also,
$$q = \frac{(\Delta t)_{\text{overall}}}{(R_{th})_{\text{total}}}$$

or
$$q = \frac{(t_{hf} - t_{cf})}{(R_{th})_{\text{conv-}hf} - R_{th-A} + R_{th-B} + R_{th-C} + (R_{th})_{\text{conv-}cf}}$$

or
$$3750 = \frac{(1250 - 25)}{\frac{1}{h_{hf}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_{cf}}} \text{ or } 3750 = \frac{1225}{\frac{1}{25} + \frac{0.25}{1.65} + \frac{0.1}{k_B} + \frac{0.15}{9.2} + \frac{1}{12}}$$

$$= \frac{1225}{0.04 + 0.1515 + \frac{0.1}{k_B} + 0.0163 + 0.0833} = \frac{1225}{0.2911 + \frac{0.1}{k_B}}$$

or
$$3750\left(0.289 + \frac{0.1}{k_B}\right) = 1225 \text{ or } \frac{0.1}{k_B} = \frac{1225}{3750} - 0.2911 = 0.0355$$

$\therefore$
$$k_B = k = \frac{0.1}{0.0355} = \mathbf{2.817\ W/m^{2}{}^\circ C.}\ \textbf{(Ans.)}$$

(*ii*) **The overall transfer coefficient, U :**

The overall heat transfer coefficient, $U = \dfrac{1}{(R_{\text{th}})_{\text{total}}}$

Now,
$$(R_{\text{th}})_{\text{total}} = \frac{1}{25} + \frac{0.25}{1.65} + \frac{0.1}{2.817} + \frac{0.15}{9.2} + \frac{1}{12}$$
$$= 0.04 + 0.1515 + 0.0355 + 0.0163 + 0.0833 = 0.3266^\circ\text{C m}^2/\text{W}$$

$\therefore$
$$U = \frac{1}{(R_{\text{th}})_{\text{total}}} = \frac{1}{0.3266} = \mathbf{3.06\ W/m^{2}{}^\circ C.}\ \textbf{(Ans.)}$$

(*iii*) **All surface temperature ; t_1, t_2, t_3, t_4 :**

$$q = q_A = q_B = q_C$$

or
$$3750 = \frac{(t_1 - t_2)}{L_A/k_A} = \frac{(t_2 - t_3)}{L_B/k_B} = \frac{(t_3 - t_4)}{L_C/k_C}$$

or
$$3750 = \frac{(1110 - t_2)}{0.25/1.65} \text{ or } t_2 = 1100 - 3750 \times \frac{0.25}{1.65} = \mathbf{531.8^\circ C}$$

Similarly,
$$3750 = \frac{(531.8 - t_3)}{0.1/2.817} \text{ or } t_3 = 531.8 - 3750 \times \frac{0.1}{2.817} = \mathbf{398.6^\circ C}$$

and
$$3750 = \frac{(398.6 - t_4)}{(0.15/9.2)} \text{ or } t_4 = 398.6 - 3750 \times \frac{0.5}{9.2} = \mathbf{337.5^\circ C}$$

[Check using outside convection, $q = \dfrac{(337.5 - 25)}{1/h_{cf}} = \dfrac{(337.5 - 25)}{1/12} = 3750 \text{ W/m}^2$]

15.2.7. Heat Conduction Through Hollow and Composite Cylinders

15.2.7.1. Heat conduction through a hollow cylinder

Refer Fig. 15.17. Consider a hollow cylinder made of material having constant thermal conductivity and insulated at both ends.

Let r_1, r_2 = Inner and outer radii ;

t_1, t_2 = Temperature of inner and outer surfaces, and

k = Constant thermal conductivity within the given temperature range.

Consider an element at radius 'r' and thickness 'dr' for a length of the hollow cylinder through which heat is transmitted. Let dt be the temperature drop over the element.

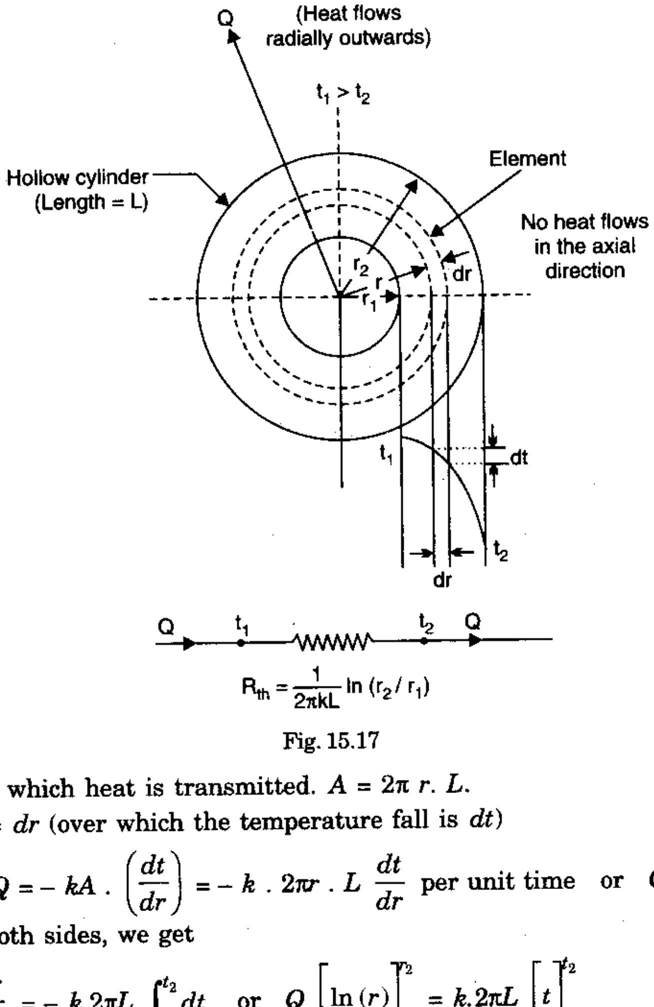


Fig. 15.17

Area through which heat is transmitted. $A = 2\pi\ r.\ L$.

Path length = dr (over which the temperature fall is dt)

$$\therefore \qquad Q = -\ kA\ .\ \left(\frac{dt}{dr}\right) = -\ k\ .\ 2\pi r\ .\ L\ \frac{dt}{dr} \text{ per unit time} \quad \text{or} \quad Q\ .\ \frac{dr}{r} = -\ k\ .\ 2\pi L.dt$$

Integrating both sides, we get

$$Q \int_{r_1}^{t_2} \frac{dr}{r} = -\ k.2\pi L \int_{t_1}^{t_2} dt \quad \text{or} \quad Q\left[\ln(r)\right]_{r_1}^{r_2} = k.2\pi L \left[t\right]_{t_1}^{t_2}$$

or $$Q.\ln(r_2/r_1) = k.2\pi L(t_2 - t_1) = k.2\pi L(t_1 - t_2)$$

$$\therefore \qquad Q = \frac{k.2\pi L(t_1 - t_2)}{\ln(r_2/r_1)} = \frac{(t_1 - t_2)}{\left[\dfrac{\ln(r_2/r_1)}{2\pi k L}\right]} \qquad ...(15.33)$$

15.2.7.2. Heat conduction through a composite cylinder

Consider flow of heat through a composite cylinder as shown in Fig. 15.18.

Let t_{hf} = The temperature of the hot fluid flowing inside the cylinder,

t_{cf} = The temperature of the cold fluid (atmospheric air),

k_A = Thermal conductivity of the inside layer A,
k_B = Thermal conductivity of the outside layer B,
t_1, t_2, t_3 = Temperature at the points 1, 2 and 3 (see Fig. 15.18),
L = Length of the composite cylinder, and
h_{hf}, h_{cf} = Inside and outside heat transfer coefficients.

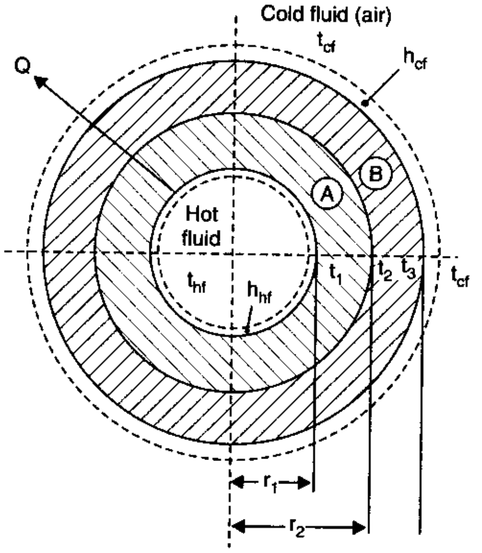


Fig. 15.18. Cross-section of a composite cylinder.

The rate of heat transfer is given by,

$$Q = h_{hf} \cdot 2\pi r_1 \cdot L(t_{hf} - t_1) = \frac{k_A \cdot 2\pi L(t_1 - t_2)}{\ln (r_2/r_1)}$$

$$= \frac{k_B \cdot 2\pi L\,(t_2 - t_3)}{\ln (r_3/r_2)} = h_{cf} \cdot 2\pi r_3 \cdot L(t_3 - t_{cf})$$

Rearranging the above expression, we get

$$t_{hf} - t_1 = \frac{Q}{h_{hf} . r_1 . 2\pi L} \quad ...(i)$$

$$t_1 - t_2 = \frac{Q}{\dfrac{k_A \cdot 2\pi L}{\ln (r_2/r_1)}} \quad ...(ii)$$

$$t_2 - t_3 = \frac{Q}{\dfrac{k_B \cdot 2\pi L}{\ln (r_3/r_2)}} \quad ...(iii)$$

$$t_3 - t_{cf} = \frac{Q}{h_{cf} \cdot r_3 \cdot 2\pi L} \quad ...(iv)$$

Adding (*i*), (*ii*), (*iii*) and (*iv*), we have

$$\frac{Q}{2\pi L}\left[\frac{1}{h_{hf} \cdot r_1} + \frac{1}{\dfrac{k_A}{\ln (r_2/r_1)}} + \frac{1}{\dfrac{k_B}{\ln (r_3/r_2)}} + \frac{1}{h_{cf} \cdot r_3}\right] = t_{hf} - t_{cf}$$

$$\therefore \quad Q = \frac{2\pi L\,(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf}.r_1} + \dfrac{1}{\dfrac{k_A}{\ln(r_2/r_1)}} + \dfrac{1}{\dfrac{k_B}{\ln(r_3/r_2)}} + \dfrac{1}{h_{cf}.r_3}\right]}$$

$$\therefore \quad Q = \frac{2\pi L\,(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf}.r_1} + \dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B} + \dfrac{1}{h_{cf}/r_3}\right]} \quad ...(15.34)$$

If there are '*n*' concentric cylinders, then

$$Q = \frac{2\pi L\,(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf}.r_1} + \displaystyle\sum_{n=1}^{n=n} \dfrac{1}{k_n} \ln\{r_{(n+1)}/r_n\} + \dfrac{1}{h_{cf}.r_{(n+1)}}\right]} \quad ...(15.35)$$

If inside the outside heat transfer coefficients are *not* considered then the above equation can be written as

$$Q = \frac{2\pi L\,[t_1 - t_{(n+1)}]}{\displaystyle\sum_{n=1}^{n=n} \dfrac{1}{k_n} \ln[r_{(n+1)}/r_n]} \quad ...(15.36)$$

Example 15.9. *A thick walled tube of stainless steel with 20 mm inner diameter and 40 mm outer diameter is covered with a 30 mm layer of asbestos insulation (k = 0.2 W/m°C). If the inside wall temperature of the pipe is maintained at 600°C and the outside insulation at 1000°C, calculate the heat loss per metre of length.* **(AMIE Summer, 2000)**

Solution. Refer Fig. 15.19,

Given, $r_1 = \frac{20}{2} = 10$ mm $= 0.01$ m

$r_2 = \frac{40}{2} = 20$ mm $= 0.02$ m

$r_3 = 20 + 30 = 50$ mm $= 0.05$ m

$t_1 = 600°C$, $t_3 = 1000°C$, $k_B = 0.2$ W/m°C

Heat transfer per metre of length, Q/L :

$$Q = \frac{2\pi L\,(t_1 - t_3)}{\dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B}}$$

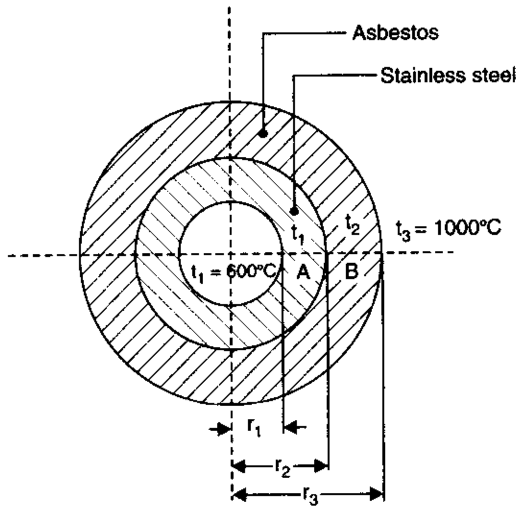


Fig. 15.19

Since the thermal conductivity of satinless steel is not given therefore, neglecting the resistance offered by stainless steel to heat transfer across the tube, we have

$$\frac{Q}{L} = \frac{2\pi(t_1 - t_3)}{\dfrac{\ln(r_3/r_2)}{k_B}} = \frac{2\pi(600 - 1000)}{\dfrac{\ln(0.05/0.02)}{0.2}} = -\ \mathbf{548.57\ W/m.}\ \ \textbf{(Ans.)}$$

Negative sign indicates that the heat transfer takes place *radially inward*.

Example 15.10. *Hot air at a temperature of 65°C is flowing through a steel pipe of 120 mm diameter. The pipe is covered with two layers of different insulating materials of thickness 60 mm and 40 mm, and their corresponding thermal conductivities are 0.24 and 0.4 W/m°C. The inside and outside heat transfer coefficients are 60 and 12 W/m°C. The atmosphere is at 20°C. Find the rate of heat loss from 60 m length of pipe.*

Solution. Refer Fig. 15.20.

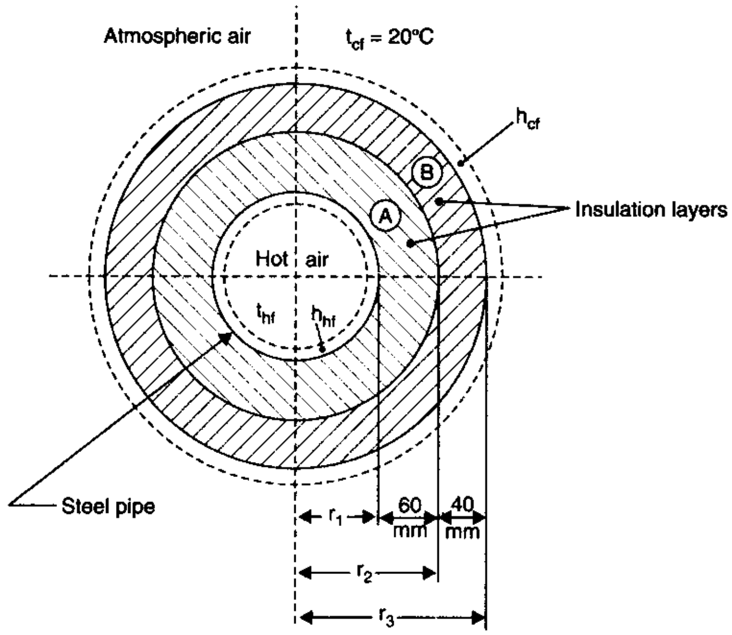


Fig. 15.20

Given : $r_1 = \frac{120}{2} = 60$ mm = 0.06 m

$r_2 = 60 + 60 = 120$ mm = 0.12 m

$r_3 = 60 + 60 + 40 = 160$ mm = 0.16 m

$k_A = 0.24$ W/m°C ; $k_B = 0.4$ W/m°C

$h_{hf} = 60$ W/m²°C ; $h_{cf} = 12$ W/m²°C

$t_{hf} = 65$°C ; $t_{cf} = 20$°C

Length of pipe, $L = 60$ m

Rate of heat loss, Q :

Rate of heat loss is given by

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} \cdot r_1} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{1}{h_{cf} \cdot r_3}\right]} \quad \text{[Eqn. (15.34)]}$$

$$= \frac{2\pi \times 60(65 - 20)}{\left[\frac{1}{60 \times 0.06} + \frac{\ln(0.12/0.06)}{0.24} + \frac{\ln(0.16/0.12)}{0.4} + \frac{1}{12 \times 0.16}\right]}$$

$$= \frac{16964.6}{0.2777 + 2.8881 + 0.7192 + 0.5208} = 3850.5 \text{ W}$$

i.e., Rate of heat loss = **3850.5 W (Ans.)**

Example 15.11. *A 150 mm steam pipe has inside dimater of 120 mm and outside diameter of 160 mm. It is insulated at the outside with asbestos. The steam temperature is 150°C and the air temperature is 20°C. h (steam side) = 100 W/m²°C, h (air side) = 30 W/m²°C, k (asbestos) = 0.8 W/m°C and k (steel) = 42 W/m°C. How thick should the asbestos be provided in order to limit the heat losses to 2.1 kW/m² ?* **(N.U.)**

Solution. Refer Fig. 15.21.

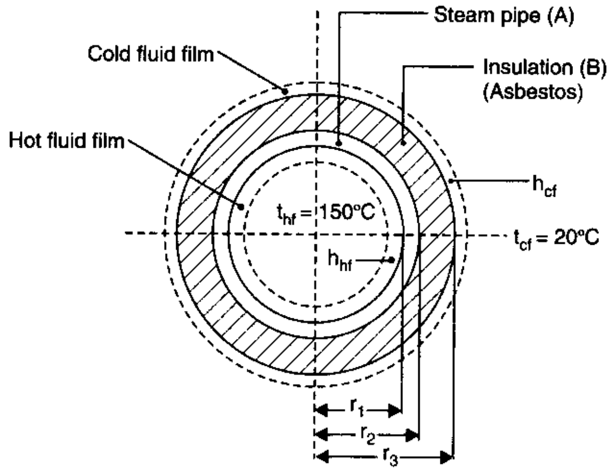


Fig. 15.21

Given :

$r_1 = \frac{120}{2} = 60 \text{ mm} = 0.06 \text{ m}$

$r_2 = \frac{160}{2} = 80 \text{ mm} = 0.08 \text{ m}$

$k_A = 42$ W/m°C ; $k_B = 0.8$ W/m°C

$t_{hf} = 150$°C ; $t_{cf} = 20$°C

$h_{hf} = 100$ W/m²°C ; $h_{cf} = 30$ W/m²°C

Heat loss = 2.1 kW/m²

Thickness of insulation (asbestos), ($r_3 - r_2$) :

Area for heat transfer = $2\pi r L$ (where L = length of the pipe)

∴ Heat loss $= 2.1 \times 2\pi r L$ kW

$= 2.1 \times 2\pi \times 0.075 \times L = 0.989 L$ kW

$= 0.989 \text{ L} \times 10^3$ watts

$\left(\text{where } r, \text{ mean radius} = \frac{150}{2} = 75 \text{ mm or } 0.075 \text{ m ... Given}\right)$

Heat transfer rate in such a case is given by

$$Q = \frac{2\pi L (t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} \cdot r_1} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{1}{h_{cf} \cdot r_3}\right]} \quad \text{...[Eqn. (15.34)]}$$

$$0.989\,L \times 10^3 = \frac{2\pi L\,(150-20)}{\left[\dfrac{1}{100\times 0.06} + \dfrac{\ln(0.08/0.06)}{42} + \dfrac{\ln(r_3/0.08)}{0.8} + \dfrac{1}{30\times r_3}\right]}$$

$$0.989 \times 10^3 = \frac{816.81}{\left[0.16666 + 0.00685 + \dfrac{\ln(r_3/0.08)}{0.8} + \dfrac{1}{30 r_3}\right]}$$

or $$\frac{\ln(r_3/0.08)}{0.8} + \frac{1}{30\,r_3} = \frac{816.81}{0.989\times 10^3} - (0.16666 + 0.00685) = 0.6524$$

or $$1.25 \ln (r_3/0.08) + \frac{1}{30 r_3} - 0.6524 = 0$$

Solving by hit and trial, we get

$$r_3 \simeq 0.105 \text{ m or } 105 \text{ mm}$$

∴ Thickness of insulation = $r_3 - r_2$ = 105 – 80 = **25 mm. (Ans.)**

15.2.8. Heat Conduction Through Hollow and Composite Spheres

15.2.8.1. Heat conduction through hollow sphere

Refer Fig. 15.22. Consider a hollow sphere made of material having constant thermal conductivity.

Let r_1, r_2 = Inner and outer radii,

t_1, t_2 = Temperature of inner and outer surfaces, and

k = Constant thermal conductivity of the material with the given temperature range.

Consider a small element of thickness dr at any radius r.

Area through which the heat is transmitted, $A = 4\pi r^2$

$$\therefore \quad Q = -k \,.\, 4\pi r^2 \,.\, \frac{dt}{dr}$$

Rearranging and integrating the above equation, we obtain

$$Q \int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi k \int_{t_1}^{t_2} dt$$

or $$Q\left[\frac{r^{-2+1}}{-2+1}\right]_{r_1}^{r_2} = -4\pi k \left[t\right]_{t_1}^{t_2}$$

or $$-Q\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = -4\pi k(t_2 - t_1)$$

or $$\frac{Q(r_2 - r_1)}{r_1 r_2} = 4\pi k\,(t_1 - t_2)$$

or $$Q = \frac{4\pi k r_1 r_2 (t_1 - t_2)}{(r_2 - r_1)} = \frac{t_1 - t_2}{\left[\dfrac{(r_2 - r_1)}{(4\pi k r_1 r_2)}\right]} \qquad ...(15.37)$$

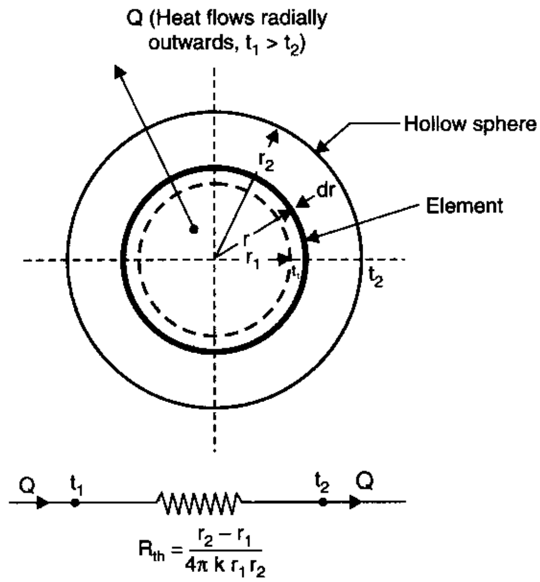


Fig. 15.22. Steady state conduction through a hollow sphere.

15.2.8.2. Heat conduction through a composite sphere

Considering Fig. 15.23 as cross-section of a composite sphere, the heat flow equation can be written as follows :

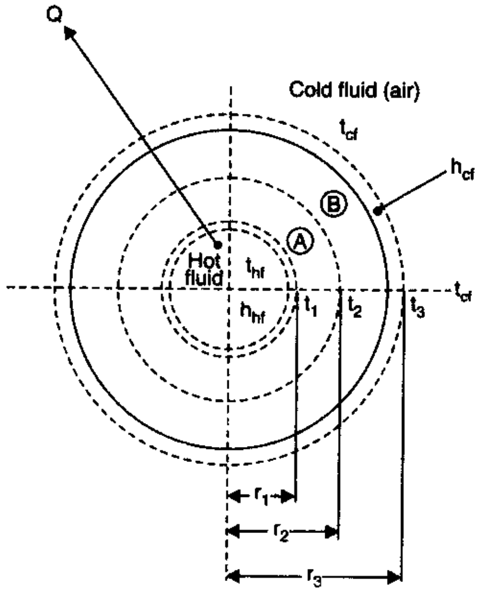


Fig. 15.23. Steady state conduction through a composite sphere.

$$Q = h_{hf} \cdot 4\pi r_1^2 \, (t_{hf} - t_1) = \frac{4\pi k_A r_1 r_2 (t_1 - t_2)}{(r_2 - r_1)} = \frac{4\pi k_B r_2 r_3 (t_1 - t_3)}{(r_3 - r_2)}$$

$$= h_{cf} \cdot 4\pi r_3^2 \, (t_3 - t_{cf})$$

By rearranging the above equation, we have

$$t_{hf} - t_1 = \frac{Q}{h_{hf} \cdot 4\pi r_1^2} \quad \text{...(i)}$$

$$t_1 - t_2 = \frac{Q (r_2 - r_1)}{4\pi k_A \cdot r_1 r_2} \quad \text{...(ii)}$$

$$t_2 - t_3 = \frac{Q (r_3 - r_2)}{4\pi k_B \cdot r_2 r_3} \quad \text{...(iii)}$$

$$t_3 - t_{cf} = \frac{Q}{h_{cf} \cdot 4\pi r_3^2} \quad \text{...(iv)}$$

Adding (*i*), (*ii*), (*iii*) and (*iv*), we get

$$\frac{Q}{4\pi}\left[\frac{1}{h_{hf} \cdot r_1^2} + \frac{(r_2 - r_1)}{k_A \cdot r_1 r_2} + \frac{(r_3 - r_2)}{k_B \cdot r_2 r_3} + \frac{1}{h_{cf} \cdot r_3^2}\right] = t_{hf} - t_{cf}$$

$$Q = \frac{4\pi (t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} \cdot r_1^2} + \frac{(r_2 - r_1)}{k_A \cdot r_1 r_2} + \frac{(r_3 - r_2)}{k_B \cdot r_2 r_3} + \frac{1}{h_{cf} \cdot r_3^2}\right]} \quad \text{...(15.38)}$$

If there are n concentric spheres then the above equation can be written as follows :

$$Q = \frac{4\pi(t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} \cdot r_1^2} + \sum_{n=1}^{n=n} \left\{\frac{r_{(n+1)} - r_n}{k_n \cdot r_n \cdot r_{(n+1)}}\right\} + \frac{1}{h_{cf} \cdot r^2_{(n+1)}}\right]} \quad ...(15.39)$$

If inside and outside heat transfer coefficients are considered, then the above equation can be written as follows :

$$Q = \frac{4\pi(t_1 - t_{(n+1)})}{\sum_{n=1}^{n=n}\left[\frac{r_{(n+1)} - r_n}{k_n \cdot r_n \cdot r_{(n+1)}}\right]} \quad ...(15.40)$$

Example 15.12. *A spherical shaped vessel of 1.4 m diameter is 90 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is 220°C. Thermal conductivity of the material of the sphere is 0.083 W/m°C.*

Solution. Refer Fig. 15.24.

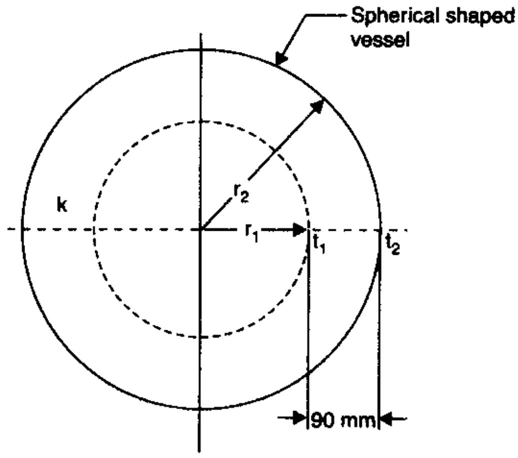


Fig. 15.24

Given : $r_2 = \frac{1.4}{2} = 0.7$ m ;

$r_1 = 0.7 - \frac{90}{1000} = 0.61$ m ;

$t_1 - t_2 = 220°C$; $k = 0.083$ W/m°C

The rate of heat transfer/leakage is given by

$$Q = \frac{(t_1 - t_2)}{\left[\frac{(r_2 - r_1)}{4\pi k r_1 r_2}\right]} \quad ...[\text{Eqn. (15.37)}]$$

$$= \frac{220}{\left[\frac{(0.7 - 0.61)}{4\pi \times 0.083 \times 0.61 \times 0.7}\right]} = 1088.67 \text{ W}$$

i.e., Rate of heat leakage = **1088.67 W. (Ans.)**

15.2.9. Critical Thickness of Insulation

15.2.9.1. Insulation-General aspects

Definition. *A material which retards the flow of heat with reasonable effectiveness* is known as *'Insulation'*. Insulation serves the following *two purposes :*

(*i*) It prevents the heat flow from the system to the surroundings ;

(*ii*) It prevents the heat flow from the surroundings to the system.

Applications :

The *fields of application* of insulations are :

(*i*) Boilers and steam pipes

(*ii*) Air-conditioning systems

(*iii*) Food preserving stores and refrigerators

(*iv*) Insulating bricks (employed in various types of furnaces)

(*v*) Preservation of liquid gases etc.

Factors affecting thermal conductivity

Some of the important factors which affect thermal conductivity (k) of the insulators (the value of k should be always low to reduce the rate of heat flow) are as follows :

1. *Temperature.* For most of the insulating materials, the value of *k increases with increase in temperature.*
2. *Density.* There is no mathematical relationship between k and ρ (density). The common understanding that high density insulating materials will have higher values of k in not always true.
3. *Direction of heat flow.* For most of the insulating materials (except few like wood) the effect of direction of heat flow on the values of k is negligible.
4. *Moisture.* It is always considered necessary to prevent ingress of moisture in the insulating materials during service, it is however difficult to find the effect of moisture on the values of k of different insulating materials.
5. *Air pressure.* It has been found that the value of *k decreases with decrease in pressure.*
6. *Convection in insulators.* The value of k increases due to the phenomenon of convection in insulators.

15.2.9.2. Critical Thickness of Insulation

The addition of insulation always increases the conductive thermal resistance. But when the total thermal resistance is made of conductive thermal resistance [$(R_{th})_{cond.}$] and convective thermal resistance [$(R_{th})_{conv.}$], the addition of insulation in *some cases* may *reduce the convective thermal resistance due to increase in surface area,* as in the case of a cylinder and a sphere, and the total thermal resistance may actually decrease resulating in increased heat flow. It may be shown that the thermal resistance actually decreases and then increases in some cases.

*"The thickness upto which heat flow increases and after which heat flow decreases is termed as **Critical thickness.*** In case of cylinders and spheres it is called ***'Critical radius'.***

A. Critical thickness of insulation for cylinder :

Consider a solid cylinder of radius r_1 insulated with an insulation of thickness $(r_2 - r_1)$ as shown in Fig. 15.25.

Let, L = Length of the cylinder,

t_1 = Surface temperature of the cylinder,

t_{air} = Temperature of air,

h_o = Heat transfer coefficient at the outer surface of the insulation, and

k = Thermal conductivity of insulating material.

Then the rate of heat transfer from the surface of the solid cylinder to the surroundings is given by

$$Q = \frac{2\pi L\,(t_1 - t_{air})}{\frac{\ln(r_2/r_1)}{k} + \frac{1}{h_o \cdot r_2}} \quad \text{...(15.41)}$$

From eqn. (15.41) it is evident that as r_2 increases, the factor $\frac{\ln(r_2/r_1)}{k}$ increases but the factor $\frac{1}{h_o \cdot r_2}$ decreases. Thus Q becomes *maximum* when the *denominator* $\left[\frac{\ln(r_2/r_1)}{k} + \frac{1}{h_o \cdot r_2}\right]$ becomes *minimum*. The required condition is

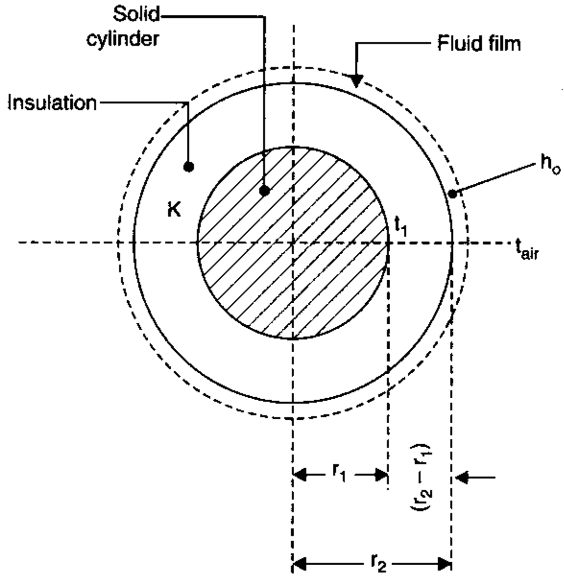


Fig. 15.25. Critical thickness of insulation for cylinder.

$$\frac{d}{dr_2}\left[\frac{\ln(r_2/r_1)}{k} + \frac{1}{h_o \cdot r_2}\right] = 0 \qquad (r_2 \text{ being the only variable})$$

$$\therefore \qquad \frac{1}{k}\cdot\frac{1}{r_2} + \frac{1}{h_o}\left(-\frac{1}{r_2^2}\right) = 0$$

or $$\frac{1}{k} - \frac{1}{h_o \cdot r_2} = 0 \qquad \text{or} \qquad h_o \cdot r_2 = k$$

or $$r_2\,(= r_c) = \frac{k}{h_o} \quad \text{...(15.42)}$$

The above relation represents the *condition for minimum resistance and consequently *maximum heat flow rate.* The *insulation radius at which resistance to heat flow is minimum* is called the '*critical radius*' (r_c). The critical radius r_c is dependent of the thermal quantities k and h_o and is independent of r_1 (*i.e.*, cylinder radius).

*It may be noted that if the second derivative of the denominator is evaluated, it will come out to be *positive*. This would verify that heat flow rate will be maximum, when $r_2 = r_c$.

In eqn. (15.41) $\ln(r_2/r_1)/k$ is the conduction (insulation) thermal resistance which *increases* with increasing r_2 and $1/h_o.r_2$ is convective thermal resistance which *decreases* with increasing r_2. At $r_2 = r_c$ *the rate of increase of conductive resistance of insulation is equal to the rate of decrease of convective resistance thus giving a minimum value for the sum of thermal resistances.*

In the physical sense we may arrive at the following conclusions :

(*i*) For cylindrical bodies with $r_1 < r_c$, the heat transfer increases by adding insulation till $r_2 = r_c$ as shown in Fig. [15.26 (*a*)]. If insulation thickness is further increased, the rate of heat loss will decrease from this peak value, but until a certain amount of insulation denoted by r_2' at b is added, the heat loss rate is still greater for the solid cylinder. This happens when r_1 is small and r_c is large, *viz.*, the thermal conductivity of the insulation k is high (poor insulating material) and h_o is low. A *practical application would be the insulation of electric cables which should be good insulator for current but poor for heat.*

(*ii*) For cylindrical bodies with $r_1 > r_c$, the heat transfer decreases by adding insulation [Fig. 15.26 (*b*)]. This happens when r_1 is large and r_c is small, *viz.*, a good insulating material is used with low k and h_o is high. In *steam and refrigeration pipes* heat insulation is the main objective. For insulation to be properly effective in restricting heat transmission, the *outer radius must be greater than or equal to the critical radius.*

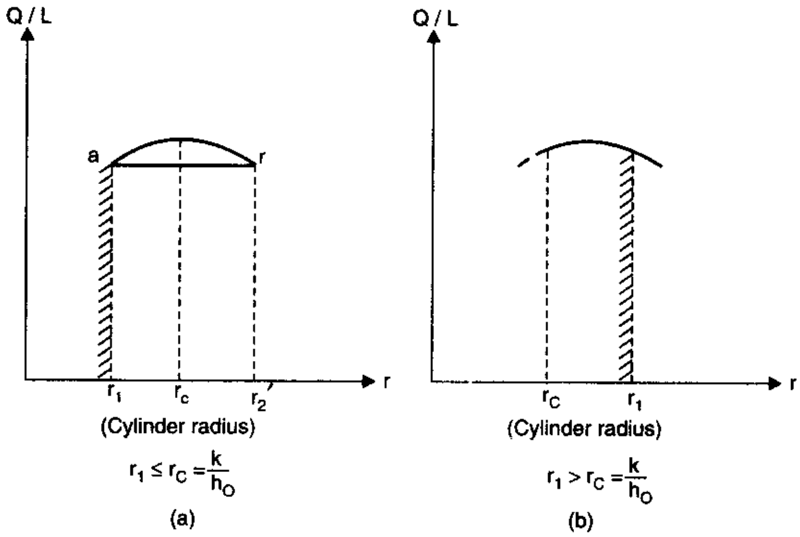


Fig. 15.26. Dependence of heat loss on insulation thickness.

B. Critical thickness of insulation for sphere :

Refer Fig. 15.27. The equation of heat flow through a sphere with insulation is given as

$$Q = \frac{(t_1 - t_{air})}{\left[\dfrac{r_2 - r_1}{4\pi k\, r_1 r_2}\right] + \dfrac{1}{4\pi\, r_2^{\,2}\, h_o}}$$

Adopting the same procedure as that of a cylinder, we have

$$\frac{d}{dr_2}\left[\frac{r_2 - r_1}{4\pi k\, r_1 r_2} + \frac{1}{4\pi r_2^{\,2} \,.\, h_o}\right] = 0$$

or $$\frac{d}{dr_2}\left[\frac{1}{kr_1} - \frac{1}{kr_2} + \frac{1}{r_2^{\,2}\, h_o}\right] = 0$$

or $$\frac{1}{kr_2^{\,2}} - \frac{2}{r_2^{\,3}\, h_o} = 0$$

or $$r_2^{\,3}\, h_o = 2kr_2^{\,2}$$

or $$r_2 (= r_c) = \frac{2k}{h_o} \qquad ...(15.43)$$

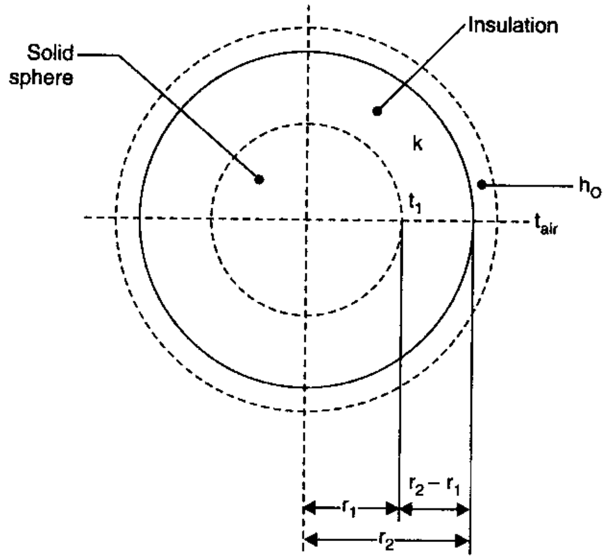


Fig. 15.27

Example 15.13. *A small electric heating application uses wire of 2 mm diameter with 0.8 mm thick insulation (k = 0.12 W/m°C). The heat transfer coefficient (h_o) on the insulated surface is 35 W/m²°C. Determine the critical thickness of insulation in this case and the percentage change in the heat transfer rate if the critical thickness is used, assuming the temperature difference between the surface of the wire and surrounding air remains unchanged.*

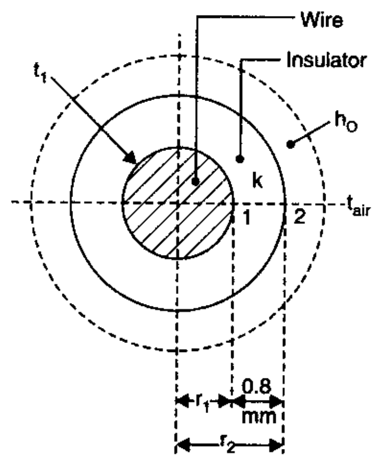


Fig. 15.28

Solution. Refer Fig. 15.28.

Given : $r_1 = \frac{2}{2} = 1$ mm = 0.001 m

$r_2 = 1 + 0.8 = 1.8$ mm = 0.0018 m

$k = 0.12$ W/m°C, $h_o = 35$ W/m²°C

Critical thickness of insulation :

The critical radius of insulation is given by

$$r_c = \frac{k}{h_o} = \frac{0.12}{35} = 3.43 \times 10^{-3} \text{ m or } 3.43 \text{ mm.}$$

∴ Critical thickness of insulation

$= r_c - r_1 = 3.43 - 1 =$ **2.43 mm. (Ans.)**

Percentage change in heat transfer rate :

Case I : The heat flow through an insulated wire is given by

$$Q_1 = \frac{2\pi L\,(t_1 - t_{air})}{\frac{\ln(r_2/r_1)}{k} + \frac{1}{h_o \cdot r_2}} = \frac{2\pi L\,(t_1 - t_{air})}{\frac{\ln(0.0018/0.001)}{0.12} + \frac{1}{35 \times 0.0018}} = \frac{2\pi L\,(t_1 - t_{air})}{20.77} \quad ...(i)$$

Case II : The heat flow through an insulated wire when critical thickness is used is given by

$$Q_2 = \frac{2\pi L (t_1 - t_{air})}{\frac{\ln(r_c / r_1)}{k} + \frac{1}{h_o \cdot r_c}} = \frac{2\pi L (t_1 - t_{air})}{\frac{\ln(0.00343/0.001)}{0.12} + \frac{1}{35 \times 0.00343}}$$

$$= \frac{2\pi L(t_1 - t_{air})}{18.6} \quad ...(ii)$$

∴ Percentage increases in heat flow by using critical thickness of insulation

$$= \frac{Q_2 - Q_1}{Q_1} \times 100 = \frac{\frac{1}{18.6} - \frac{1}{20.77}}{\frac{1}{20.77}} \times 100 = \textbf{11.6\%}. \quad \textbf{(Ans.)}$$

15.3. HEAT TRANSFER BY CONVECTION

- The rate equation for the convective heat transfer (regardless of particular nature) between a surface and an adjacent fluid is prescribed by *Newton's law of cooling* (Refer Fig. 15.29)

$$Q = hA(t_s - t_f) \quad ...(15.44)$$

where, Q = Rate of conductive heat transfer,

A = Area exposed to heat transfer,

t_s = Surface temperature,

t_f = Fluid temperature, and

h = Co-efficient of conductive heat transfer.

The units of h are, $h = \frac{Q}{A(t_s - t_f)} = \frac{\text{W}}{\text{m}^{2\circ}\text{C}}$ or W/m²°C or W/m²K

The coefficient of convective heat transfer 'h' (also known as film heat transfer coefficient) may be defined as *"the amount of heat transmitted for a unit temperature difference between the fluid and unit area of surface in unit time."*

The value of 'h' depends on the following factors :

(*i*) Thermodynamic and transport properties (*e.g.*, viscosity, density, specific heat etc.) ;

(*ii*) Nature of fluid flow ;

(*iii*) Geometry of the surface ;

(*iv*) Prevailing thermal conditions.

Since 'h' depends upon several factors, it is difficult to frame a single equation to satisfy all the variations, however a dimensional analysis gives an equation for the purpose which is given as under :

$$\frac{h_D}{k} = Z\left(\frac{\rho CD}{\pi}\right)^a \left(\frac{c_p\mu}{k}\right)^b \left(\frac{D}{L}\right)^c \quad ...(15.45)$$

or

$$Nu = Z\,(Re)^a\,(Pr)^b \left(\frac{D}{L}\right)^c$$

where, Nu = Nusselt number $\left(\frac{h_D}{k}\right)$,

Re = Reynolds number $\left(\frac{\rho \bar{u} D}{\mu}\right)$,

Pr = Prandtl number $\left(\frac{c_p \mu}{k}\right)$,

$\frac{D}{L}$ = Diameter to length ratio,

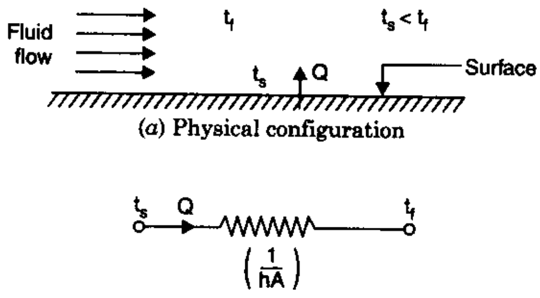
Z = A constant to be determined experimentally,

ρ = Density,

μ = Dynamic viscosity, and

C = Velocity.

The *mechanisms of convection* in which phase changes are involved lead to the important *fields of boiling and condensation.*



(*a*) Physical configuration

(*b*) Equivalent circuit

Fig. 15.29. Convective heat-transfer.

Refer Fig. 15.29 (*b*). The quantity $\frac{1}{hA}\left[Q = \frac{t_s - t_f}{(1/hA)} \ldots \text{Eqn}\,(28.44)\right]$ is called *convection thermal resistance* $[(R_{th})_{conv.}]$ *to heat flow.*

- **Dimensionless numbers :**

Reynolds numbers, $Re = \frac{VL}{\nu}$

Prandtl number, $Pr = \frac{\mu c_p}{k} = \frac{\nu}{\alpha}$

Nusselt number, $Nu = \frac{hL}{k}$

Stanton number, $St = \frac{h}{\rho V c_p} = \frac{Nu}{Re \times Pr}$

Peclet number, $Pe = \frac{LV}{\alpha} (= Re.Pr)$

Graetz number, $G = Pe\left(\frac{\pi D}{4}\right)$

Grashoff number, $Gr = \frac{\rho^2 \beta g \Delta t L^3}{\mu^2}$.

Example 15.14. *A hot plate 1 m × 1.5 m is maintained at 300°C. Air at 25°C blows over the plate. If the convective heat transfer coefficient is 20 W/m²°C, calculate the rate of heat transfer.*

Solution. Area of the plate exposed to heat transfer, $A = 1 \times 1.5 = 1.5$ m²

Plate surface temperature, t_s = 300°C

Temperature of air (fluid), t_f = 20°C

Convective heat-transfer coeffficient, h = 20 W/m²°C

Rate of heat transfer, Q :

From Netwon's law of cooling,

$$Q = hA\,(t_s - t_f)$$

$$= 20 \times 1.5(300 - 20) = 8400 \text{ W or } \mathbf{8.4\ kW}. \quad \textbf{(Ans.)}$$

Example 15.15. *A wire 1.5 mm in diameter and 150 mm long is submerged in water at atmospheric pressure. An electric current is passed through the wire and is increased until the water boils at 100°C. Under the condition if convective heat transfer coefficient is 4500 W/m²°C find how much electric power must be supplied to the wire to maintain the wire surface at 120°C ?*

Solution. Diameter of the wire, d = 1.5 mm = 0.0015 m

Length of the wire, l = 150 mm = 0.15 m

$\therefore$ Surface area of the wire (exposed to heat transfer),

$$A = \pi\, d\, l = \pi \times 0.0015 \times 0.15 = 7.068 \times 10^{-4}\ \text{m}^2$$

Wire surface temperature, t_s = 120°C

Water temperature, t_f = 100°C

Convective heat transfer coefficient, h = 4500 W/m²°C

Electric power to be supplied :

Electric power which must be supplied = Total convection loss (Q)

$\therefore$ $Q = hA(t_s - t_f) = 4500 \times 7.068 \times 10^{-4}\ (120 - 100) =$ **63.6 W. (Ans.)**

Example 15.16. *Water flows inside a tube 45 mm in diameter and 3.2 m long at a velocity of 0.78 m/s. Determine the heat transfer co-efficient and the rate of heat transfer if the mean water temperature is 50°C and the wall is isothermal at 70°C. For water at 50°C take k = 0.66 W/mK, ν = 0.478 × 10⁻⁶ m²/s and Prandtl number = 2.98.*

Solution. Diameter of the tube, D = 45 mm = 0.045 m

Length of the tube, l = 3.2 m

Velocity of water, $\bar{u}$ = 0.78 m/s

For water at 60°C, k = 0.66 W/mK

Kinematic viscosity, $\nu = 0.478 \times 10^{-6}$ m²/s

Pr = 2.98

Reynolds number is given by

$$Re = \frac{D\bar{u}}{\nu} = \frac{0.045 \times 0.78}{0.478 \times 10^{-6}} = 73431$$

From Dittus and Boelter equation, Nusselt number,

$$Nu = 0.023\ (Re)^{0.8}\ (Pr)^{0.4}$$

$$\frac{hD}{k} = 0.023\ (73431)^{0.8}\ (2.98)^{0.4}$$

$$\frac{h \times 0.045}{0.66} = 0.023 \times 7810.9 \times 1.547$$

$$\therefore \quad h = 4076\ \text{W/m}^2\ \text{K}$$

i.e., Heat transfer co-efficient = **4076 W/m² K (Ans).**

$$Q = hA\ (t_w - t_f)$$
$$= 4076 \times \pi DL\ (70 - 50)$$
$$= 4076 \times \pi \times 0.045 \times 3.2 \times 20 = 36878 \text{ or } 36.878\ \text{kW}$$

i.e., Rate of heat transfer = **36.878 kW. (Ans).**

Example 15.17. *When 0.5 kg of water per minute is passed through a tube of 20 mm diameter, it is found to be heated from 20°C to 50°C. The heating is accomplished by condensing steam on the surface of the tube and subsequently the surface temperature of the tube is maintained at 85°C. Determine the length of the tube required for developed flow.*

Take the thermo-physical properties of water at 60°C as :

ρ = 983.2 kg/m², c_p = 4.178 kJ/kg K, k = 0.659 W/m°C, ν = 0.478 × 10⁻⁶ m²/s.

Solution. *Given :* m = 0.5 kg/min, D = 20 mm = 0.02 m, t_i = 20°C, t_o = 50°C

Length of the tube required for fully developed flow, L :

The mean film temperature, $t_f = \dfrac{1}{2}\left(85 + \dfrac{20 + 50}{2}\right) = 60°\text{C}$

Let us first determine the type of the flow

$$m = \rho A\bar{u} = 983.2 \times \frac{\pi}{4} \times (0.02)^2 \times \bar{u} = \frac{0.5}{60} \text{ (kg/s)}$$

or
$$\bar{u} = \frac{0.5}{60} \times \frac{4}{\pi} \times \frac{1}{983.2 \times (0.02)^2} = 0.0269 \text{ m/s}$$

Reynolds number, $Re = \dfrac{D.\bar{u}}{\nu} = \dfrac{0.02 \times 0.0269}{0.478 \times 10^{-6}} = 1125.5$

Since $Re < 2000$, hence the flow is *laminar.*

With *constant wall temperature* having *fully developed flow,*

$$Nu = \frac{hD}{k} = 3.65$$

or
$$h = \frac{3.65\,k}{D} = \frac{3.65 \times 0.659}{0.02} = 120.26 \text{ W/m}^2{}^\circ\text{C}$$

The rate of heat transfer, $Q = A_s\, h\, (t_s - t_\infty) = m\, c_p\, (t_o - t_i)$

Here, $t_\infty = \dfrac{20 + 50}{2} = 35°\text{C} = t_b$

$$\therefore \quad (\pi \times 0.02 \times L) \times 120.26 \times (85 - 35) = \frac{0.5}{60} \times (4.178 \times 10^3)(50 - 20)$$

or
$$377.8\, L = 1044.5$$

or
$$L = \frac{1044.5}{377.8} = \mathbf{2.76\ m.} \quad \textbf{(Ans.)}$$

15.4. HEAT EXCHANGERS

15.4.1. Introduction

A **'heat exchanger'** may be defined *as an equipment which transfers the energy from a hot fluid to a cold fluid, with maximum rate and minimum investment and running costs.*

In heat exchangers the temperature of each fluid changes as it passes through the exchangers, and hence the temperature of the dividing wall between the fluids also changes along the length of the exchanger.

Examples of heat exchangers :

(*i*) Intercoolers and preheaters ;
(*ii*) Condensers and boilers in steam plant ;
(*iii*) Condensers and evaporators in refrigeration units ;
(*iv*) Regenerators ;
(*v*) Automobile radiators ;
(*vi*) Oil coolers of heat engine ;
(*vii*) Milk chiller of a pasteurising plant ;
(*viii*) Several other industrial processes.

15.4.2. Types of Heat Exchangers

In order to meet the widely varying applications, several types of heat exchangers have been developed which are classified on the basis of *nature of heat exchange process, relative direction of fluid motion, design and constructional features,* and *physical state of fluids.*

1. Nature of heat exchange process

Heat exchangers, on the basis of nature of heat exchange process, are classified as follows :

(*i*) Direct contact (or open) heat exchangers.

(*ii*) Indirect contact heat exchangers.

(*a*) Regenerators.
(*b*) Recuperators.

(*i*) ***Direct contact heat exchangers.*** In a direct contact or open heat exchanger the exchange of heat takes place by direct mixing of hot and cold fluids and transfer of heat and mass takes place simultaneously. The use of such units is made under conditions where mixing of two fluids is either harmless or desirable. *Examples* : (*i*) Cooling towers ; (*ii*) Jet condensers ; (*iii*) Direct contact feed heaters.

Fig. 15.30 shows a direct contact heat exchanger in which steam mixes with cold water, gives its latent heat to water and gets condensed. Hot water and non-condensable gases leave the container as shown in the figure.

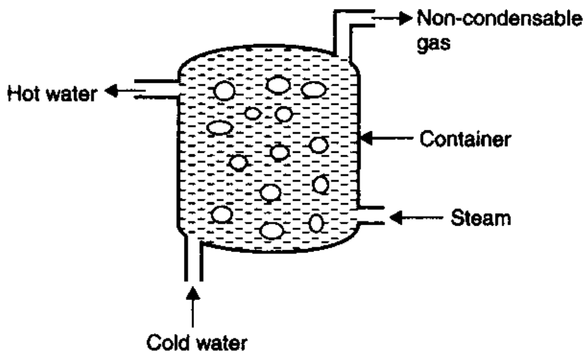


Fig. 15.30. Direct contact or open heat exchanger.

(*ii*) ***Indirect contact heat exchangers.*** In this type of heat exchanger, the heat transfer between two fluids could be carried out by transmission through wall which separates the two fluids. This type includes the following :

(*a*) Regenerators.

(*b*) Recuperators or surface exchangers.

(*a*) *Regenerators* : In a *regenerator* type of heat exchanger the *hot and cold fluids pass alternately through a space containing solid particles (matrix), these particles providing alternately a sink and a source for heat flow.*

Examples : (*i*) I.C. engines and gas turbines ; (*ii*) Open hearth and glass melting furnaces ; (*iii*) Air heaters of blast furnaces.

A *regenerator* generally operates periodically (the solid matrix alternately stores heat extracted from the hot fluid and then delivers it to the cold fluid). However, in some regenerators the matrix is made to rotate through the fluid passages arranged side by side which makes the heat exchange process *continuous.*

The performance of these regenerators is affected by the following *parameters :*

(*i*) Heat capacity of regenerating material,

(*ii*) The rate of absorption, and (*iii*) The release of heat.

Advantages :

1. Higher heat transfer coefficient ;
2. Less weight per kW of the plant ;
3. Minimum pressure loss ;
4. Quick response to load variation ;
5. Small bulk weight ;
6. Efficiency quite high.

Disadvantages :

1. Costlier compared to recuperative heat exchangers.
2. Leakage is the main trouble, therefore, perfect sealing is required.

(*b*) *Recuperators : 'Recuperator'* is the most important type of heat exchanger in which the flowing fluids exchanging heat are on either side of *dividing wall* (in the form of pipes or tubes generally). These heat exchangers are used when two fluids cannot be allowed to mix *i.e.,* when the mixing is undesirable.

Examples : (*i*) Automobile radiators, (*ii*) Oil coolers, intercoolers, *air preheaters,* economisers, superheaters, condensers and surface feed heaters of a steam power plant, (*iii*) Milk chiller of pasteurising plant, (*iv*) Evaporator of an ice plant :

Advantages :

1. Easy construction ;
2. More economical ;
3. More surface area for heat transfer ;
4. Much suitable for stationary plants.

Disadvantages :

1. Less heat transfer coefficient ;
2. Less generating capacity ;
3. Heavy and sooting problems.

The flow through *direct heat exchangers and recuperators* may be treated as *steady state* while through regenerators the *flow is essentially transient.*

2. Relative direction of fluid motion

According to the relative directions of two fluid streams the heat exchangers are classified into the following *three* categories :

(*i*) Parallel-flow or unidirection flow (*ii*) Counte-flow (*iii*) Cross-flow.

(*i*) ***Parallel-flow heat exchangers.*** In a *parallel-flow exchanger,* as the name suggests, the two fluid streams (hot and cold) travel in the *same direction.* The two streams enter at one end and leave at the other end. The flow arrangement and variation of temperatures of the fluid streams in case of parallel flow heat exchangers, are shown in Fig. 15.31. It is evident from the Fig. 15.31. (*b*) that the *temperature difference* between the hot and cold fluids goes on *decreasing* from inlet to outlet. Since this type of heat exchanger needs a large area of heat transfer, therefore, it is *rarely used in practice.*

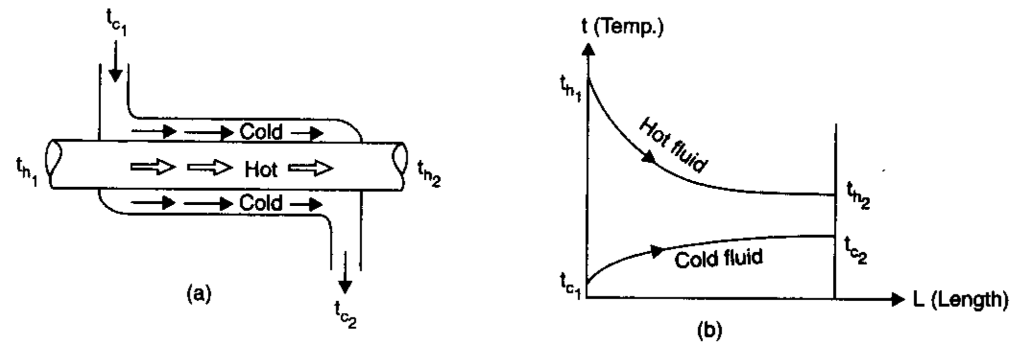


Fig. 15.31. Parallel-flow heat exchanger.

Examples : Oil coolers, oil heaters, water heaters etc.

As the two fluids are separated by a wall, this type of heat exchanger may be called *parallel-flow recuperator or surface heat exchanger.*

(*ii*) ***Counter-flow heat exchangers.*** In a counter-flow heat exchanger, the two fluids flow in *opposite directions.* The hot and cold fluids enter at the opposite ends. The flow arrangement and temperature distribution for such a heat exchanger are shown schematically in Fig. 15.32. The *temperature difference* between the two fluids remains more or less *nearly constant.* This type of heat exchanger, due to counter flow, gives *maximum rate of heat transfer for a given surface area.* Hence such heat exchangers are *most favoured* for heating and cooling of fluids.

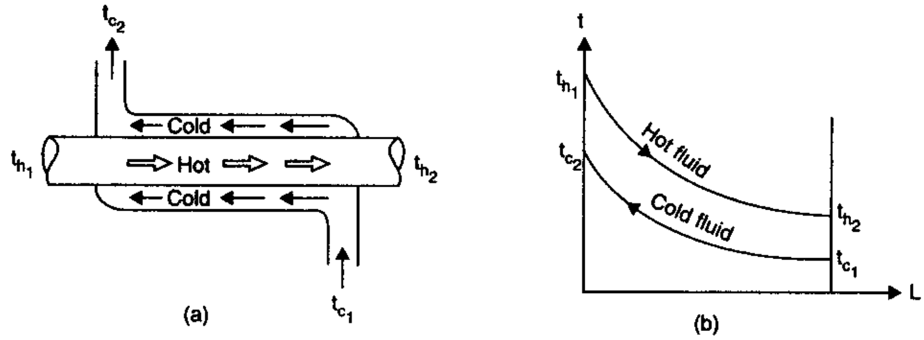


Fig. 15.32. Counter-flow heat exchanger.

(*iii*) ***Cross-flow heat exchangers.*** In cross-flow heat exchangers, the *two fluids (hot and cold) cross one another in space, usually at right angles.* Fig. 15.32 shows a schematic diagram of common arrangements of cross-flow heat exchangers.

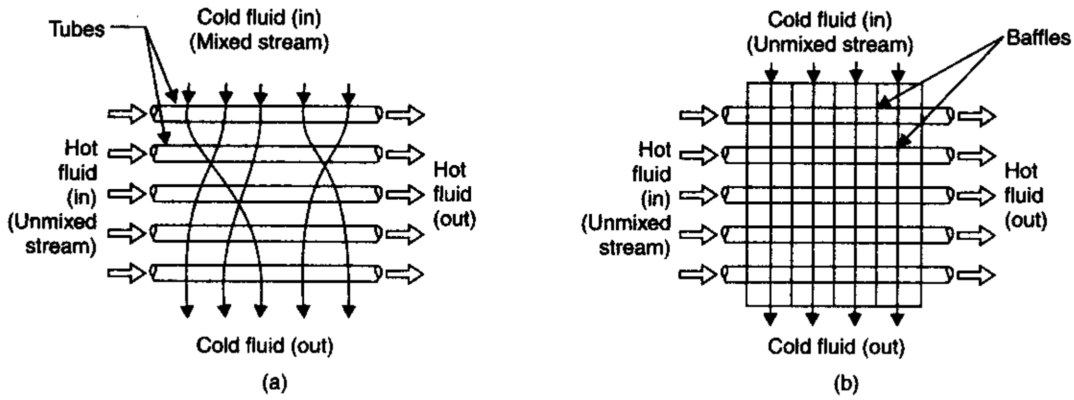


Fig. 15.33. Cross-flow heat exchangers.

- Refer Fig. 15.33 (*a*) : *Hot fluid* flows in the separate tubes and there is no mixing of the fluid streams. The *cold fluid* is perfectly mixed as it flows through the exchanger. The temperature of this mixed fluid will be uniform across any section and will vary only in the direction of flow.

Examples : The cooling unit of refrigeration system etc.

- Refer Fig. 15.33 (*b*) : In this case each of the fluids follows a prescribed path and is unmixed as it flows through heat exchanger. Hence the temperature of the fluid leaving the heater section is not uniform.

Examples : Automobile radiator etc.

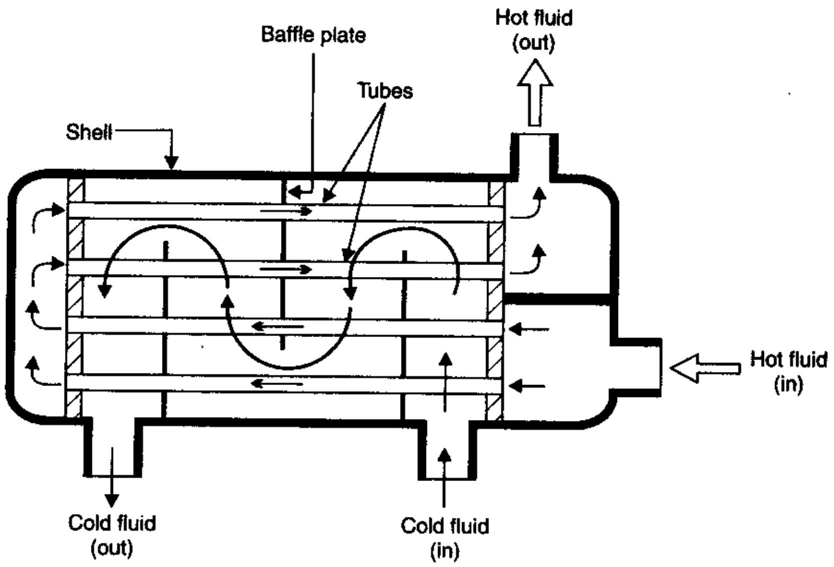
- In yet another arrangement, *both the fluids are mixed* while they travel through the exchanger ; consequently the temperature of both the fluids is uniform across the section and varies only in the direction in which flow takes place.

3. Design and constructional features

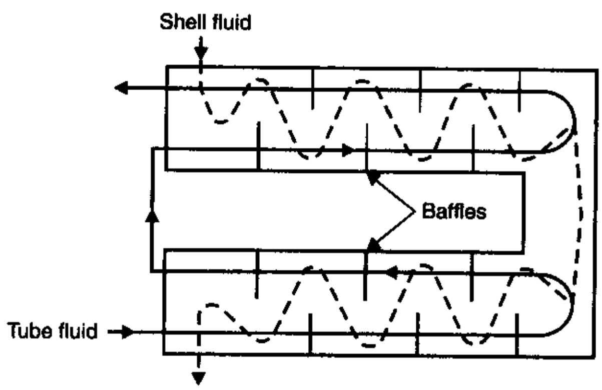
On the basis of design and constructional features, the heat exchangers are classified as under :

(*i*) ***Concentric tubes.*** In this type, two concentric tubes are used, each carrying one of the fluids. This direction of flow may be parallel or counter as depicted in Figs. 15.31 (*a*) and Fig. 15.32 (*a*). The effectiveness of the heat exchanger is increased by using swirling flow.

(*ii*) ***Shell and tube.*** In this type of heat exchanger one of the fluids flows through a bundle of tubes enclosed by a shell. The other fluid is forced through the shell and it flows over the outside surface of the tubes. Such an arrangement is employed where *reliability* and *heat transfer effectiveness are important.* With the use of multiple tubes heat transfer rate is amply improved due to increased surface area.



(*a*) One-shell pass and two-tube pass heat exchanger.



(*b*) Two-shell pass and Four-tube pass heat exchanger

Fig. 15.34. Shell and tube heat exchangers.

(*iii*) ***Multiple shell and tube passes.*** Multiple shell and tube passes are used for *enhancing the overall heat transfer. Multiple shell pass* is possible where the fluid flowing through the shell is re-routed. The shell side fluid is forced to flow back and forth across the tubes by baffles. *Multiple tube pass* exchangers are those which re-route the fluid through tubes in the opposite direction.

(*iv*) ***Compact heat exchangers.*** These are special purpose heat exchangers and have a very large transfer surface area per unit volume of the exchanger. They are generally employed when convective heat transfer sufficient associated with one of the fluids is much smaller than that associated with the other fluid.

Example : Plate-fin, flattened fin tube exchangers etc.

4. Physical state of fluids

Depending upon the physical state of fluids the heat exchangers are classified as follows :

(*i*) Condensers (*ii*) Evaporators

(*i*) ***Condensers.*** In a condenser, the condensing fluid remains at constant temperature throughout the exchanger while the temperature of the colder fluid gradually increases from inlet to outlet. The hot fluid loses latent part of heat which is accepted by the cold fluid (Refer Fig. 15.35).

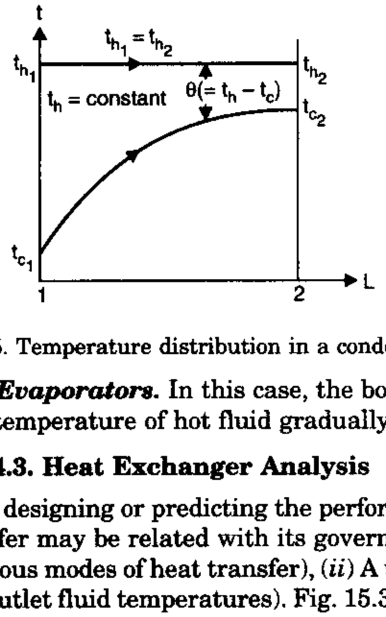


Fig. 15.35. Temperature distribution in a condenser.

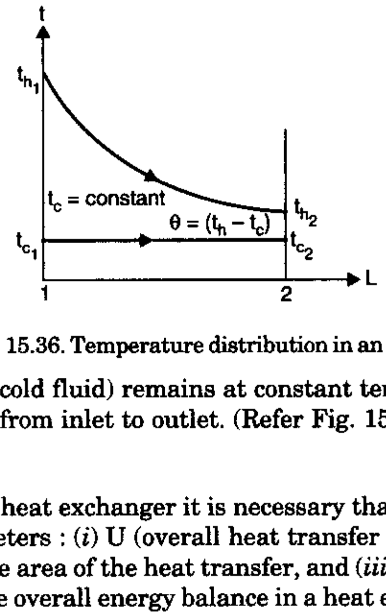


Fig. 15.36. Temperature distribution in an evaporator.

(*ii*) ***Evaporators.*** In this case, the boiling fluid (cold fluid) remains at constant temperature while the temperature of hot fluid gradually decreases from inlet to outlet. (Refer Fig. 15.36).

15.4.3. Heat Exchanger Analysis

For designing or predicting the performance of a heat exchanger it is necessary that the total heat transfer may be related with its governing parameters : (*i*) U (overall heat transfer coefficient due to various modes of heat transfer), (*ii*) A total surface area of the heat transfer, and (*iii*) t_1, t_2 (the inlet and outlet fluid temperatures). Fig. 15.37 shows the overall energy balance in a heat exchanger.

Let, $\dot{m}$ = Mass flow rate, kg/s,

c_p = Specific heat of fluid at constant pressure J/kg°C,

t = Temperature of fluid, °C, and

Δt = Temperature drop or rise of a fluid across the heat exchanger.

Subscripts *h* and *c* refer to the *hot* and *cold* fluids respectively ; subscripts 1 and 2 correspond to the *inlet* and *outlet* conditions respectively.

Assuming that there is no heat loss to the surroundings and potential and kinetic energy changes are negligible, from the energy balance in a heat exchanger, we have :

Heat given up by the hot fluid, $Q = m_h \, c_{ph} \, (t_{h_1} - t_{h_2})$ ...(15.46)

Heat picked up by the cold fluid, $Q = m_c \, c_{pc} \, (t_{c_2} - t_{c_1})$ ...(15.47)

Total heat transfer rate in the heat exchanger, $Q = UA\ \theta_m$ ...(15.48)

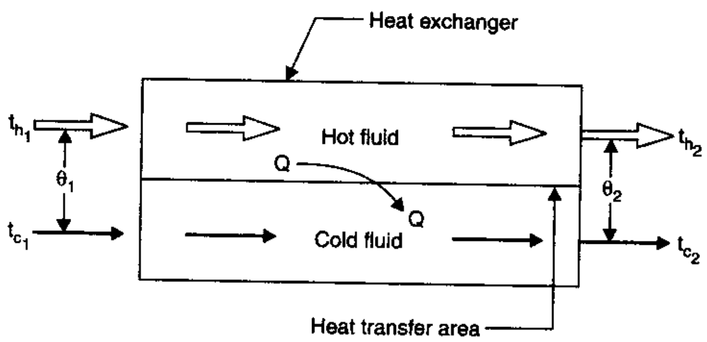


Fig. 15.37. Overall energy balance in a heat exchanger.

where, U = Overall heat transfer coefficient between the two fluids,

A = Effective heat transfer area, and

θ_m = *Appropriate mean value of temperature difference or logarithmic mean temperature difference (LMTD).*

15.4.4. Logarithmic Mean Temperature Difference (LMTD)

Logarithmic mean temperature difference (LMTD) is defined as *that temperature difference which, if constant, would give the same rate of heat transfer as actually occurs under variable conditions of temperature difference.*

In order to derive expression for *LMTD* for various types of heat exchangers, the following ***assumptions*** are made :

1. The overall heat transfer coefficient U is constant.
2. The flow conditions are steady.
3. The specific heats and mass flow rates of both fluids are constant.
4. There is no loss of heat to the surroundings, due to the heat exchanger being perfectly insulated.
5. There is no change of phase either of the fluids during the heat transfer.
6. The changes in potential and kinetic energies are negligible.
7. Axial conduction along the tubes of the heat exchanger is negligible.

15.4.4.1. Logarithmic Mean Temperature Difference for "Parallel-flow"

Refer Fig. 15.38, which shows the flow arrangement and distribution of temperature in a single-pass parallel-flow heat exchanger.

Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through this elementary area is given by

$$dQ = U\,dA\,(t_h - t_c) = U\,.\,dA\,.\,\Delta t$$

As a result of heat transfer dQ through the area dA, the hot fluid is cooled by dh whereas the cold fluid is heated up by dt_c. The energy balance over a differential area dA may be written as

$$dQ = -\,\dot{m}_h\,.\,c_{ph}\,.\,dt_h = \dot{m}_c\,.\,c_{pc}\,.\,dt_c = U\,.\,dA\,.\,(t_h - t_c) \qquad ...(15.49)$$

(Here dt_h is – ve and dt_c is + ve)

or
$$dt_h = -\frac{dQ}{\dot{m}_h c_{ph}} = -\frac{dQ}{C_h}$$

and $$dt_c = \frac{dQ}{\dot{m}_c c_{pc}} = \frac{dQ}{C_c}$$

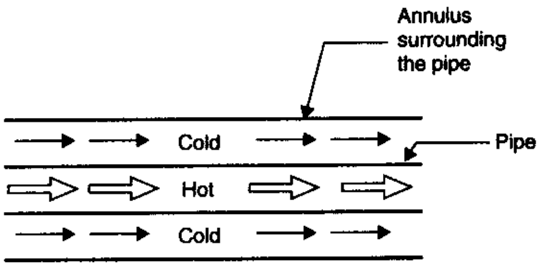
where, $C_h = \dot{m}_h\ c_{ph}$ = Heat capacity or water equivalent of hot fluid, and

$C_c = \dot{m}_c\ c_{pc}$ = Heat capacity or water equivalent of cold fluid.

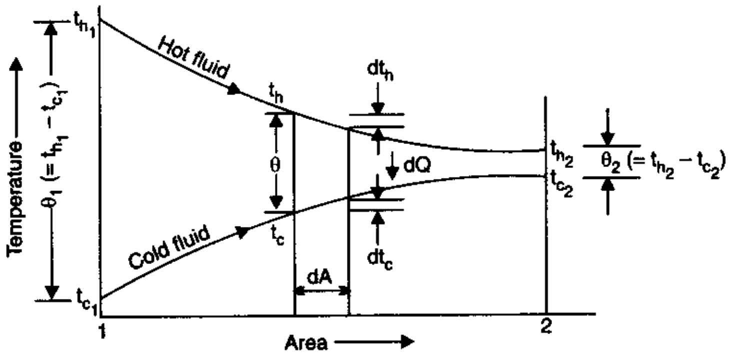
$\dot{m}_h$ and $\dot{m}_c$ are the mass flow rates of fluids and c_{ph} and c_{pc} are the respective specific heats.

$$\therefore \quad dt_h - dt_c = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$d\theta = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \quad ...(15.50)$$



(a) Flow arrangement.



(b) Temperature distribution.

Subscripts *h, c* refer to *hot* and *cold* fluids
Subscripts 1, 2 refer to *inlet* and *outlet* conditions.
Fig. 15.38. Calculation of LMTD for a parallel-flow heat exchanger.

Substituting the value of dQ from eqn. (15.49) the above equation becomes

$$d\theta = -U\,.\,dA\,(t_h - t_c)\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

or $$d\theta = -U\,.\,dA\,.\,\theta\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

or $$\frac{d\theta}{\theta} = -U.\,dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Integrating between inlet and outlet conditions (*i.e.*, from $A = 0$ to $A = A$), we get

$$\int_1^2 \frac{d\theta}{\theta} = -\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \int_{A=0}^{A=A} U \,.\, dA$$

or

$$\ln(\theta_2/\theta_1) = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \quad ...(15.51)$$

Now, the total heat transfer rate between the two fluids is given by

$$Q = C_h\,(t_{h_1} - t_{h_2}) = C_c\,(t_{c_2} - t_{c_1}) \quad ...(15.52)$$

or

$$\frac{1}{C_h} = \frac{t_{h_1} - t_{h_2}}{Q} \quad ...[15.52\ (a)]$$

$$\frac{1}{C_c} = \frac{t_{c_2} - t_{c_1}}{Q} \quad ...[15.52\ (b)]$$

Substituting the values of $\frac{1}{C_h}$ and $\frac{1}{C_c}$ into eqn. (15.51), we get

$$\ln(\theta_2/\theta_1) = -UA\left[\frac{t_{h_1} - t_{h_2}}{Q} + \frac{t_{c_2} - t_{c_1}}{Q}\right]$$

$$= \frac{UA}{Q}\,[(t_{h_2} - t_{c_2}) - (t_{h_1} - t_{c_1})] = \frac{UA}{Q}\,(\theta_2 - \theta_1)$$

$$Q = \frac{UA\,(\theta_1 - \theta_1)}{\ln(\theta_2/\theta_1)}$$

The above equation may be written as

$$Q = U A\,\theta_m \quad ...(15.53)$$

where

$$\theta_m = \frac{\theta_2 - \theta_1}{\ln(\theta_2/\theta_1)} = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} \quad ...(15.54)$$

θ_m is called the *logarithmic mean temperature difference (LMTD).*

15.4.4.2. Logarithmic Mean Temperature Difference for "Counter-flow"

Ref Fig. 15.39, which shows the flow arrangement and temperature distribution in a single-pass counter-flow heat exchanger.

Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through this elementary area is given by

$$dQ = U \,.\, dA\,(t_h - t_c) = U \,.\, dA \,.\, \Delta t \quad ...(15.55)$$

In this case also, due to heat transfer dQ through the area dA, the hot fluid is cooled down by dt_h whereas the cold fluid is heated by dt_c. The energy balance over a differential area dA may be written as

$$dQ = -\dot{m}_h \,.\, c_{ph} \,.\, dt_h = -\dot{m}_c \,.\, c_{pc} \,.\, dt_c \quad ...(15.56)$$

In a counter-flow system, the temperatures of both the fluids *decrease* in the direction of heat exchanger length, hence the –ve signs.

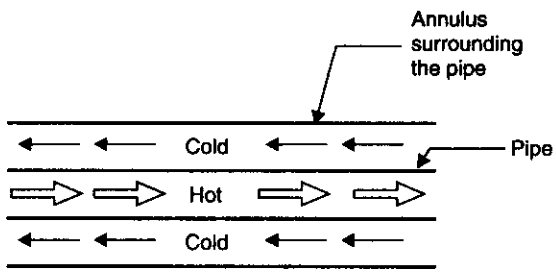
∴

$$dt_h = -\frac{dQ}{\dot{m}_h c_{ph}} = -\frac{dQ}{C_h}$$

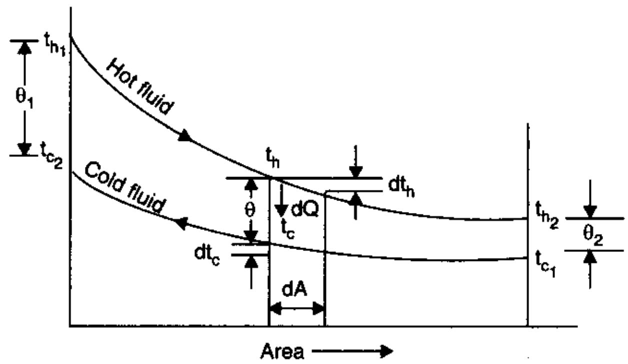
and

$$dt_c = -\frac{dQ}{\dot{m}_c c_{pc}} = -\frac{dQ}{C_c}$$

$$dt_h - dt_c = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$



(*a*) Flow arrangement.



(*b*) Temperature distribution.

Fig. 15.39. Calculation of LMTD for a counter-flow heat exchanger.

or $$d\theta = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right] \qquad ...(15.57)$$

Inserting the value of dQ from eqn. (15.55), we get

$$d\theta = -U\,dA\,(t_h - t_c)\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$= -U\,dA\,.\,\theta\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

or $$\frac{d\theta}{\theta} = -U\,dA\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

Integrating the above equation from $A = 0$ to $A = A$, we get

$$\ln(\theta_2/\theta_1) = -U\,.\,A\left[\frac{1}{C_h} - \frac{1}{C_c}\right] \qquad ...(15.58)$$

Now, the total heat transfer rate between the two fluids is given by

$$\theta = C_h\,(t_{h_1} - t_{h_2}) = C_c\,(t_{c_2} - t_{c_1}) \qquad ...(15.59)$$

or $$\frac{1}{C_h} = \frac{t_{h_1} - t_{h_2}}{Q} \qquad ...[15.60\ (a)]$$

or $$\frac{1}{C_c} = \frac{t_{c_2} - t_{c_1}}{Q} \qquad ...[15.60\ (b)]$$

Substituting the values of $\frac{1}{C_h}$ and $\frac{1}{C_c}$ into eqn. (15.58), we get

$$\ln(\theta_2/\theta_1) = -UA\left[\frac{t_{h_1}-t_{h_2}}{Q} - \frac{t_{c_2}-t_{c_1}}{Q}\right]$$

$$= -\frac{UA}{Q}[(t_{h_1}-t_{c_2})-(t_{h_2}-t_{c_1})] = -\frac{UA}{Q}(\theta_1-\theta_2) = \frac{UA}{Q}(\theta_2-\theta_1)$$

or
$$Q = \frac{UA(\theta_2-\theta_1)}{\ln(\theta_2/\theta_1)} \quad \text{...(15.61)}$$

Since
$$Q = UA\,\theta_m$$

$\therefore$
$$\theta_m = \frac{\theta_2-\theta_1}{\ln(\theta_2/\theta_1)} = \frac{\theta_1-\theta_2}{\ln(\theta_1/\theta_2)} \quad \text{...(15.61(a))}$$

A special case arises when $\theta_1 = \theta_2 = \theta$ in case of a *counter-flow* heat exchanger. In such a case, we have

$$\theta_m = \frac{\theta-\theta}{\ln(\theta/\theta)} = \frac{0}{0}$$

This value is indeterminate. The value of θ_m for such can be found by applying L' Hospital's rule :

$$\lim_{\theta_2\to\theta_1}\frac{\theta_2-\theta_1}{\ln(\theta_2/\theta_1)} = \lim_{(\theta_2/\theta_1)\to 1}\frac{\theta_1\left[\frac{\theta_2}{\theta_1}-1\right]}{\ln(\theta_2/\theta_1)}$$

Let $(\theta_2/\theta_1) = R$. Therefore, the above expression can be written as

$$\lim_{R\to 1}\frac{\theta(R-1)}{\ln(R)}$$

Differentiating the numerator and denominator with respect to R and taking limits, we get

$$\lim_{(R\to 1)}\frac{\theta}{(1/R)} = \theta$$

Hence when $\theta_1 = \theta_2$ eqn. (15.61) becomes

$$Q = UA\,\theta$$

θ_m *(LMTD) for a counter-flow unit is always greater than that for a parallel-flow unit* ; hence counter-flow heat exchanger can transfer *more* heat than parallel-flow one ; in other words a counter-flow heat exchanger needs a *smaller heating surface for the same rate of heat transfer.* For this reason, the counter-flow arrangement is usually used.

When the temperature variations of the fluids are relatively small, then temperature variation curves are approximately straight lines and adequately accurate results are obtained by taking the *arithmetic mean temperature difference (AMTD).*

$$AMTD = \frac{t_{h_1}+t_{h_2}}{2} - \frac{t_{c_1}+t_{c_2}}{2} = \frac{(t_{h_1}-t_{c_1})+(t_{h_2}-t_{c_2})}{2} = \frac{\theta_1+\theta_2}{2} \quad \text{...(15.62)}$$

However, practical considerations suggest that the logarithmic mean temperature difference (θ_m) should be invariably used when $\frac{\theta_1}{\theta_2} > 1.7$.

Example 15.18. *The flow rates of hot and cold water streams running through a parallel-flow heat exchanger are 0.2 kg/s and 0.5 kg/s respectively. The inlet temperatures on the hot and cold sides are 75°C and 20°C respectively. The exit temperature of hot water is 45°C. If the individual heat transfer coefficients on both sides are 650 W/m²°C, calculate the area of the heat exchanger.*

Solution. *Given :* $\dot{m}_h$ = 0.2 kg/s ; $\dot{m}_c$ = 0.5 kg/s ; t_{h_1} = 75°C ;

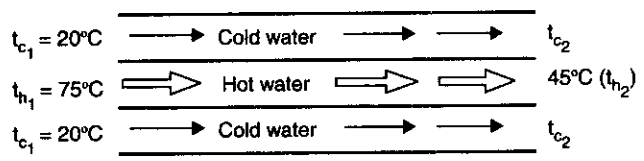
t_{h_2} = 45°C ; t_{c_1} = 20°C ; $h_i = h_o$ = 650 W/m²°C.

The area of heat exchanger, A :

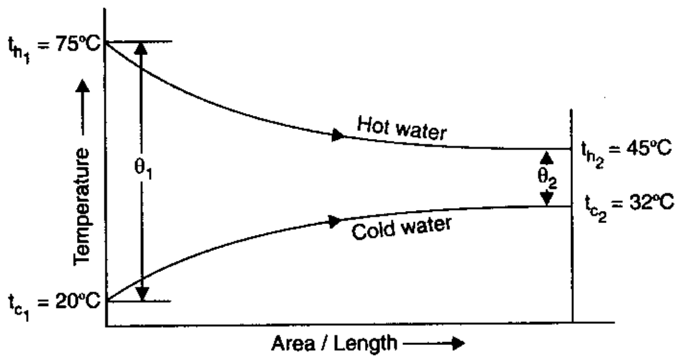
The heat exchanger is shown diagrammatically in Fig. 15.40.

The heat transfer rate, $Q = m_h \times c_{ph} \times (t_{h_1} - t_{h_2})$

$= 0.2 \times 4.187 \times (75 - 45) = 25.122$ kJ/s



(*a*) Flow arrangement.



(*b*) Temperature distribution.

Fig. 15.40. Parallel-flow heat exchanger.

Heat lost by hot water = Heat gained by cold water

$$\dot{m}_h \times c_{ph} \times (t_{h_1} - t_{h_2}) = \dot{m}_c \times c_{pc} \times (t_{c_2} - t_{c_1})$$

$$0.2 \times 4.187 \times (75 - 45) = 0.5 \times 4.187 \times (t_{c_2} - 20)$$

$\therefore$ t_{c_2} = 32°C

Logarithmic mean temperature difference (*LMTD*) is given by,

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} \quad \text{...[Eqn. (15.54)]}$$

or

$$\theta_m = \frac{(t_{h_1} - t_{c_1}) - (t_{h_2} - t_{c_2})}{\ln[(t_{h_1} - t_{c_1})/(t_{h_2} - t_{c_2})]}$$

$$= \frac{(75 - 20) - (45 - 32)}{\ln[(75 - 20)/(45 - 32)]} = \frac{55 - 33}{\ln(55/13)} = 29.12°\text{C}$$

Overall heat transfer coefficient U is calculated from the relation

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{650} + \frac{1}{650} = \frac{1}{325}$$

$\therefore$ U = 325 W/m²°C

Also, $Q = U A \theta_m$

or $$A = \frac{Q}{U\theta_m} = \frac{25.122 \times 1000}{325 \times 29.12} = \mathbf{2.65\ m^2}. \quad \textbf{(Ans.)}$$

Example 15.19. *In a counter-flow double pipe heat exchanger, water is heated from 25°C to 65°C by an oil with a specific heat of 1.45 kJ/kg K and mass flow rate of 0.9 kg/s. The oil is cooled from 230°C to 160°C. If the overall heat transfer coefficient is 420 W/m²°C, calculate the following :*

(*i*) *The rate of heat transfer,*

(*ii*) *The mass flow rate of water, and*

(*iii*) *The surface area of the heat exchanger.*

Solution. *Given :* t_{c_1} = 25°C ; t_{c_2} = 65°C, c_{ph} = 1.45 kJ/kg K ; $\dot{m}_h$ = 0.9 kg/s ;

t_{h_1} = 230°C ; t_{h_2} = 160°C, U = 420 W/m²°C.

(*i*) **The rate of heat transfer, *Q* :**

$$Q = \dot{m}_h \times c_{ph} \times (t_{h_1} - t_{h_2})$$

or $$Q = 0.9 \times (1.45) \times (230 - 160) = \mathbf{91.35\ kJ/s}$$

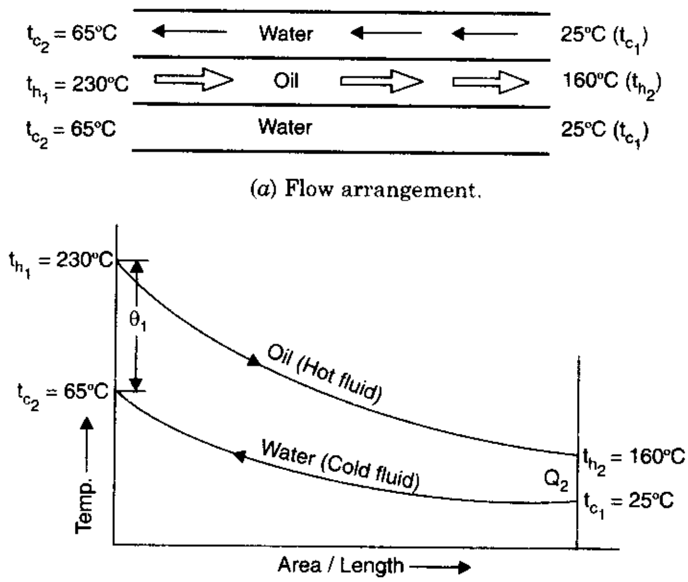
(*ii*) **The mass flow rate of water, $\dot{m}_c$:**

Heat lost by oil (hot fluid) = Heat gained by water (cold fluid)

$$\dot{m}_h \times c_{hp} \times (t_{h_1} - t_{h_2}) = \dot{m}_c \times c_{pc} \times (t_{c_2} - t_{c_1})$$

$$91.35 = \dot{m}_c \times 4.187\,(65 - 25)$$

∴ $$\dot{m}_c = \frac{91.35}{4.187 \times (65 - 25)} = \mathbf{0.545\ kg/s}$$



(*a*) Flow arrangement.

(*b*) Temperature distribution.

Fig. 15.41. Counter-flow heat exchanger.

(*iii*) **The surface area of heat exchanger, *A* :**

Logarithmic mean temperature difference (LMTD) is given by

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)}$$

$$= \frac{(t_{h_1} - t_{c_2}) - (t_{h_2} - t_{c_1})}{\ln [(t_{h_1} - t_{c_2})/(t_{h_2} - t_{c_1})]} = \frac{(230 - 65) - (160 - 25)}{\ln [(230 - 65)/(160 - 25)]}$$

or $$\theta_m = \frac{165 - 135}{\ln [(165/135)]} = 149.5°\text{C}$$

Also, $$Q = U A \theta_m$$

or $$A = \frac{Q}{U\theta_m} = \frac{91.35 \times 10^3}{420 \times 149.5} = \mathbf{1.45\ m^2.} \quad \textbf{(Ans.)}$$

Example 15.20. *Steam enters a counter-flow heat exchanger, dry saturated at 10 bar and leaves at 35°C. The mass flow of steam is 800 kg/min. The gas enters the heat exchanger at 650°C and mass flow rate is 1350 kg/min. If the tubes are 30 mm diameter and 3 m long, determine the number of tubes required. Neglect the resistance offered by metallic tubes. Use the following data :*

For steam : t_{sat} = 180°C (at 10 bar) ; c_{ps} = 2.71 kJ/kg°C ; h_s = 600 W/m²°C

For gas : c_{pg} = 1 kJ/kg°C ; h_g = 250 W/m²°C **(P.U.)**

Solution. *Given* : $\dot{m}_s = \dot{m}_c = \frac{800}{60} = 13.33$ kg/s ; $\dot{m}_g = \dot{m}_h = \frac{1350}{60} = 22.5$ kg/s ;

t_{h_1} = 650°C ; $t_{c_1}(= t_{sat})$ = 180°C ; t_{c_2} = 350°C ; d = 30 mm = 0.03 m ; L = 3 m.

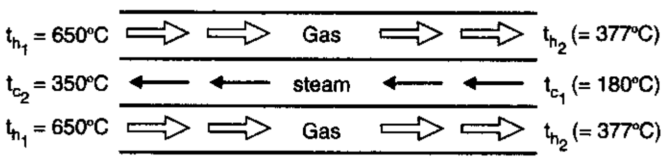
Number of tubes required, *N* :

Heat lost by gases = Heat gained by steam

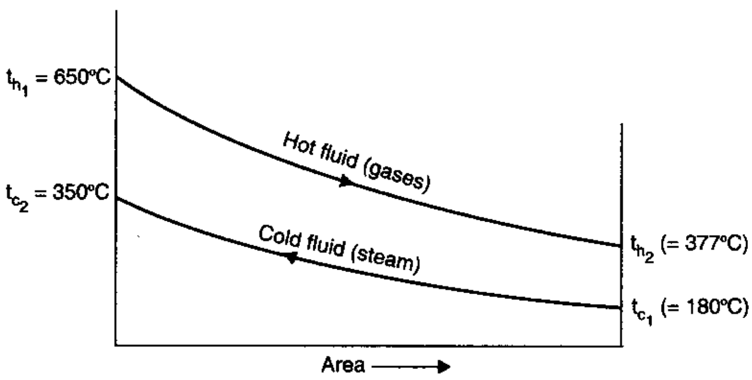
$$\dot{m}_h \times c_{ph} \times (t_{h_1} - t_{h_2}) = \dot{m}_c \times c_{pc} \times (t_{c_2} - t_{c_1})$$

$$22.5 \times 1 \times (650 - t_{h_2}) = 13.33 \times 2.71 \times (350 - 180)$$

$$\therefore \quad t_{h_2} = 377°\text{C}$$



(*a*) Flow arrangement.



(*b*) Temperature distribution.

Fig. 15.42. Counter-flow heat exchanger.

Overall heat transfer coefficient is given by

$$\frac{1}{U} = \frac{1}{h_g} + \frac{d_o}{d_i}\frac{1}{h_s} = \frac{1}{h_g} + \frac{1}{h_s} \text{ as } d_i \approx d_o \quad \text{...(given)}$$

or

$$U = \frac{h_g \times h_s}{h_g + h_s} = \frac{250 \times 600}{250 + 600} = 176.5 \text{ W/m}^2\text{°C}$$

Total heat transfer rate is given by

$$Q = U A \theta_m \quad \text{...}(i)$$

where $A = N \times (\pi d L) = N \times \pi \times 0.03 \times 3 = 0.2827 \text{ N m}^2$

$Q = 22.5 \times (1 \times 10^3) \times (650 - 377) = 6142.5 \times 10^3$ W

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} = \frac{(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})}{\ln[(t_{h1} - t_{c2})/(t_{h2} - t_{c1})]}$$

$$= \frac{(650 - 350) - (377 - 180)}{\ln[(650 - 300)/(377 - 180)]} = \frac{300 - 197}{\ln(300/197)} = 244.9\text{°C}$$

Substituting the values is eqn. (*i*), we get

$$6142.5 \times 10^3 = 176.5 \times 0.2827 \text{ N} \times 244.9$$

or

$$N = \frac{6142.5 \times 10^3}{176.5 \times 0.2827 \times 244.9} = \textbf{503 tubes.} \quad \textbf{(Ans.)}$$

Example 15.21. *A two-pass surface condenser is required to handle the exhaust from a turbine developing 15 MW with specific steam consumption of 5 kg/kWh. The condenser vacuum is 660 mm of Hg when the barometer reads 760 mm of Hg. The mean velocity of water is 3 m/s, water inlet temperature is 24°C. The condensate is saturated water and outlet temperature of cooling water in 4°C less than the condensate temperature. The quality of exhaust steam is 0.9 dry. The overall heat transfer coefficient based on outer area of tubes is 4000 W/m²°C. The water tubes are 38.4 mm in outer diameter and 29.6 mm in inner diameter. Calculate the following :*

(*i*) *Mass of cooling water circulated in kg/min,*

(*ii*) *Condenser surface area,*

(*iii*) *Number of tubes required per pass, and*

(*iv*) *Tube length.* **(P.U.)**

Solution. *Given* : d_i = 29.6 mm = 0.0296 m ; d_o = 38.4 mm = 0.0384 m ;

U = 4000 W/m²°C ; V = 3 m/s ; t_{c_1} = 24°C ; x (dryness fraction) = 0.9.

The pressure of the steam in the condenser,

$$p_s = \frac{760 - 660}{760} \times 1.0133 = 0.133 \text{ bar}$$

The properties of steam at p_s = 0.133 bar, from steam table, are :

$$t_{sat} = 51\text{°C} ; h_{fg} = 2592 \text{ kJ/kg}$$

$$\therefore \quad t_{c_1} = 51 - 4 = 47\text{°C}$$

The steam condensed per minute,

$$\dot{m}_s (= \dot{m}_h) = \frac{(15 \times 1000) \times 5}{60} = 1250 \text{ kg/min}$$

(*i*) **Mass of cooling water circulated per minute, $\dot{m}_w$ (= $\dot{m}_c$) :**

Heat lost by steam = Heat gained by water

$$\dot{m}_h \times (x \,.\, h_{fg}) = \dot{m}_c \times c_{pc} \times (t_{c_2} - t_{c_1})$$

$$1250 \times (0.9 \times 2592) = \dot{m}_c \times 4.187\ (47 - 24)$$

$$\therefore \quad \dot{m}_c\ (= \dot{m}_w) = \mathbf{30280\ kg/min}$$

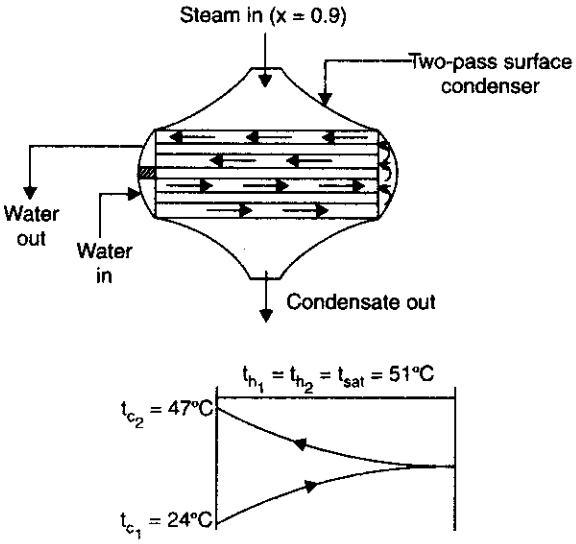


Fig. 15.43. A two-pass surface condenser.

(*ii*) **Condenser surface area, *A* :**

$$Q = \frac{\dot{m}_s \times (x\,.\,h_{fg})}{60} = U A\,\theta_m \quad ...(i)$$

where,
$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} = \frac{(t_{h_1} - t_{c_1}) - (t_{h_2} - t_{c_2})}{\ln[(t_{h_1} - t_{c_1})/(t_{h_2} - t_{c_2})]}$$

$$= \frac{(51-24)-(51-47)}{\ln[(51-24)/(51-47)]} = \frac{(27-4)}{\ln(27/4)} = 12.04°C$$

Substituting the values in eqn. (*i*), we get

$$\frac{1250}{60} \times (0.9 \times 2592 \times 10^3) = 4000 \times A \times 12.04$$

or
$$A = \mathbf{1009.1\ m^2}$$

(*iii*) **Number of tubes required per pass, N_p :**

$$\dot{m}_w = \left(\frac{\pi}{4} d_i^2 \times V \times \rho\right) \times N_p$$

$$\frac{30280}{60} = \frac{\pi}{4} \times (0.0296)^2 \times 3 \times 1000 \times N_p$$

or
$$N_p = \frac{30280 \times 4}{60 \times \pi \times (0.0296)^2 \times 3 \times 1000} = 244.46 \text{ say } \mathbf{245}$$

(Total number of tubes required, $N = 2N_p = 2 \times 245 = 490$)

(*iv*) **Tube length, L :**

$$A = (\pi d_o L) \times (2N_p)$$

$$1009.1 = (\pi \times 0.0384 \times L) \times (2 \times 245)$$

or $$L = \frac{1009.1}{\pi \times 0.0384 \times 2 \times 245} \simeq \textbf{17.1 m.} \quad \textbf{(Ans.)}$$

Example 15.22. *A feed water heater which supplies hot water to a boiler comprises a shell and tube heat exchanger with one-shell pass and two-tube passes. One hundred thin-walled tubes each of 20 mm diameter and length of 2 m per pass are used. Under normal operating conditions, water enters the tubes at 10 kg/s and 17°C and is heated by condensing saturated steam at 1 atm. on the outer surface of the tubes. The convection coefficient of the saturated steam is 10 kW/m²°C. Determine the water exit temperature.*

Use the following properties of water :

$c_p = 4.18$ *kJ/kg°C* ; $\mu = 0.596 \times 10^{-3}$ *Ns/m²* ; $k = 0.635$ *W/m°C and* $Pr = 3.93$. **(M.U.)**

Solution. *Given* : p (number of tube passes) = 2, N (total number of tubes) = 200,

d = 20 mm = 0.02 m ; (length per pass) = 2 m, $\dot{m}_w = \dot{m}_c$ = 10 kg/s, t_{c_1} = 17°C.

Water exit temperature, t_{c_2} :

$$\dot{m}_c = \frac{\pi}{4} d^2 \times V \times \rho \times N_p$$

$$\left[\text{where } V = \text{velocity of water} ; N_p = \text{number of tubes per pass} = \frac{N}{p} = \frac{200}{2} = 100\right]$$

or $$10 = \frac{\pi}{4} \times 0.02^2 \times V \times 1000 \times 100$$

$\therefore$ $$V = \frac{10 \times 4}{\pi \times 0.02^2 \times 1000 \times 100} = 0.318 \text{ m/s}$$

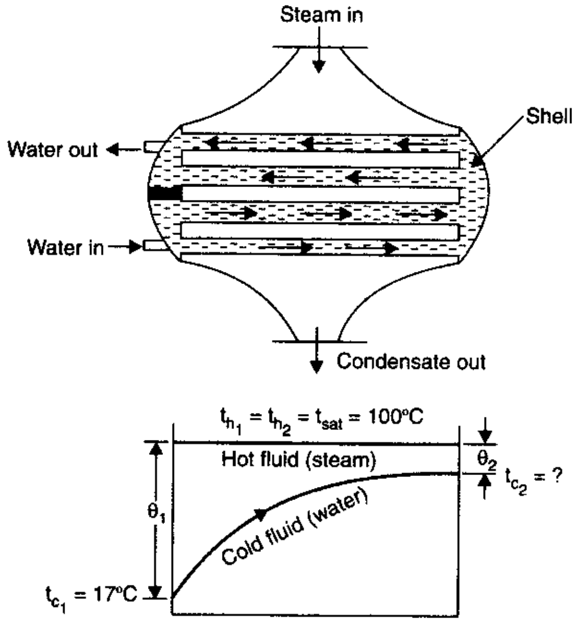


Fig. 15.44. One-shell pass and two-tube passes condenser.

Using non-dimensional heat transfer equation to water side, we get

$$Nu = \frac{h_i d}{k} = 0.023 \, (Re)^{0.8} \, (Pr)^{0.33}$$

or $$h_i = \frac{k}{d} \times 0.023\, (Re)^{0.8}\, (Pr)^{0.33} \quad ...(i)$$

$$Re = \frac{\rho V d}{\mu} = \frac{1000 \times 0.318 \times 0.02}{0.596 \times 10^{-3}} = 10671$$

Substituting the values in eqn. (*i*), we get

$$h_i = \frac{0.635}{0.02} \times 0.023\, (10671)^{0.8}\, (3.93)^{0.33} = 1915 \text{ W/m}^2\text{°C}$$

The overall heat transfer coefficient is given by the relation,

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$\frac{1}{U} = \frac{1}{1915} + \frac{1}{10 \times 10^3} = 0.000622$$

$\therefore$ $$U = \frac{1}{0.000622} = 1607.7 \text{ W/m}^2\text{°C}$$

Further, $$\theta_1 = t_{h_1} - t_{c_1} = 100 - 17 = 83\text{°C}$$

$$\theta_2 = t_{h_2} - t_{c_2} = 100 - t_{c_2}$$

$\therefore$ Arithmetic mean temperature difference,

$$AMTD = \frac{\theta_1 + \theta_2}{2} = \frac{83 + (100 - t_{c_2})}{2} = 91.5 - 0.5\, t_{c_2}$$

The heat transfer rate is given by,

$$Q = \dot{m}_c\, c_{pc}\, (t_{c_2} - t_{c_1}) = U A_s\, (AMTD) = U \times (\pi\, d\, L \times N)\, (AMTD)$$

(where A_s = Surface area of *all the tubes in both passes*)

or $$10 \times (4.18 \times 10^3)\, (t_{c_2} - 17) = 1607.7 \times (\pi \times 0.02 \times 2 \times 200) \times (91.5 - 0.5\, t_{c_2})$$

$$41800\, (t_{c_2} - 17) = 40406\, (91.5 - 5\, t_{c_2})$$

or $$t_{c_2} - 17 = \frac{40406}{41800}\, (91.5 - 0.5\, t_{c_2}) = 0.966\, (91.5 - 0.5\, t_{c_2})$$

$$= 88.39 - 0.483\, t_{c_2}$$

or $$t_{c_2} = \mathbf{71°C.} \quad \textbf{(Ans.)}$$

15.5. HEAT TRANSFER BY RADIATION

15.5.1. Introduction

'Radiation' heat transfer is defined as *"the transfer of energy across a system boundary by means of an electromagnetic mechanism which is caused solely by a temperature difference."* Whereas the heat transfer by conduction and convection takes place only in the *presence of medium, radiation heat transfer does not require a medium.* Radiation exchange, in fact, occurs most effectively in *vacuum.* Further, the rate of heat transfer by *conduction and convection* varies as the temperature difference to the *first power,* whereas the *radiant heat exchange* between two bodies depends on the difference between their temperature to the *'fourth power'.* Both the amount of radiation and the quality of radiation depend upon temperature. The *dissipation from the filament of a vacuum tube* or the *heat leakage through the evacuated walls of a thermos flask* are some familiar *examples* of heat transfer by radiation.

The contribution of *radiation* to heat transfer is very significant at high absolute temperature levels such as those prevailing in *furnaces, combustion chambers, nuclear explosions* and in *space applications.* The *solar energy* incident upon the earth is also governed by the *laws of radiation.*

The energy which a radiating surface releases is *not continuous* but is in the form of successive and separate (discrete) packet or quanta of energy called *photons*. The photons are propagated through space as rays ; the movement of swarm of photons is described as *electromagnetic waves*. The photons travel (with speed equal to that of light) in straight paths with unchanged frequency ; when they approach the receiving surface, there occurs reconversion of wave motion into *thermal energy* which is partly absorbed, reflected or transmitted through the receiving surface (the magnitude of each fraction depends, upon the nature of the surface that receives the *thermal radiation*).

All types of electromagnetic waves are classified in terms of *wavelength* and are propagated at the speed of light (c) *i.e.*, 3×10^8 m/s. The electromagnetic spectrum is shown in Fig. 15.45. The distinction between one form of radiation and another lies only in its frequency (f) and wavelength (λ) which are related by

$$c = \lambda \times f \quad ...(15.63)$$

The emission of thermal radiation (range lies between wavelength of 10^{-7} m and 10^{-4} m) *depends upon the nature, temperature and state of the emitting surface.* However, with *gases the dependence is also upon the thickness of the emitting layer and the gas pressure.*

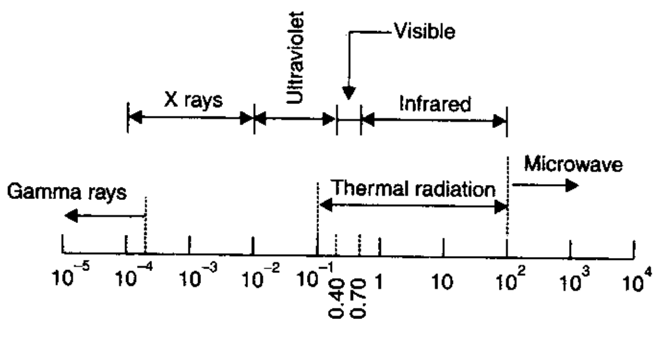


Fig. 15.45. Spectrum of electromagnetic radiation.

Thermal radiations exhibit characteristics similar to those of *visible light*, and *follow optical laws*. These can be *reflected, refracted* and are *subject to scattering and absorption when they pass through a media*. They get *polarised* and *weakened* in strength with inverse square of radial distance from the radiating surface.

15.5.2. Surface Emission Properties

The rate of emission of radiation by a body depends upon the following *factors* :

(*i*) The temperature of the surface,

(*ii*) The nature of the surface, and

(*iii*) The wavelength or frequency of radiation.

The parameters which deal with the surface emission properties are given below :

(*i*) **Total emissive power (E).** The *emissive power* is defined as the *total amount of radiation emitted by a body per unit area and time*. It is expressed in W/m². The *emissive power of a black body*, according to Stefan- Boltzmann, is *proportional to absolute temperature to the fourth power.*

$$E_b = \sigma T^4 \text{ W/m}^2 \quad ...(15.64)$$

$$E_b = \sigma A T^4 \text{ W} \quad ...[15.64\ (a)]$$

where σ = Stefan-Boltzmann constant = 5.67×10^{-8} W/m² K⁴.

(*ii*) **Monochromatic (spectral) emissive power (E_1).** It is often necessary to determine the spectral distribution of the energy radiated by a surface. At any given temperature the amount of radiation emitted per unit wavelength varies at different wavelengths. For this purpose the *monochromatic emissive power* E_λ of the surface is used. It is defined as *the rate of energy radiated per unit area of the surface per unit wavelength.*

The total emissive power is given by

$$E = \int_0^\infty E_\lambda \, d\lambda \ \text{W/m}^2 \qquad ...(15.65)$$

(*iii*) **Emission from real surface-emissivity.** The emissive power from a real surface is given by

$$E = \varepsilon \, \sigma \, AT^4 \ \text{W} \qquad ...(15.66)$$

where ε = emissivity of the material.

Emissivity(ε). It is defined as the *ability of the surface of a body to radiate heat.* It is also defined as the *ratio of the emissive power of any body to the emissive power of a black body of equal temperature* $\left(i.e., \varepsilon = \dfrac{E}{E_b}\right)$. Its values varies for different substances ranging from 0 to 1. For a black body $\varepsilon = 1$, for a white body surface $\varepsilon = 0$ and for gray bodies it lies between 0 and 1. It may vary with temperature or wavelength.

(*iv*) *Intensity of radiation.*

(*v*) *Radiation density and pressure.*

(*vi*) *Radiosity* (*J*). It refers to *all of the radiant energy leaving a surface.*

(*vii*) *Interrelationship between surface emission and irradiation properties.*

15.5.3. Absorptivity, Reflectivity and Transmissivity

When incident radiation also called **irradiation** (defined as the *total incident radiation on a surface from all directions per unit time and per unit area of surface),* expressed in W/m^2 and denoted by (G) impinges on a surface, three things happens ; a part is *reflected* back (G_r), a part is *transmitted* through (G_t), and the remainder is *absorbed* (G_a) depending upon the characteristics of the body, as shown in Fig. 15.46.

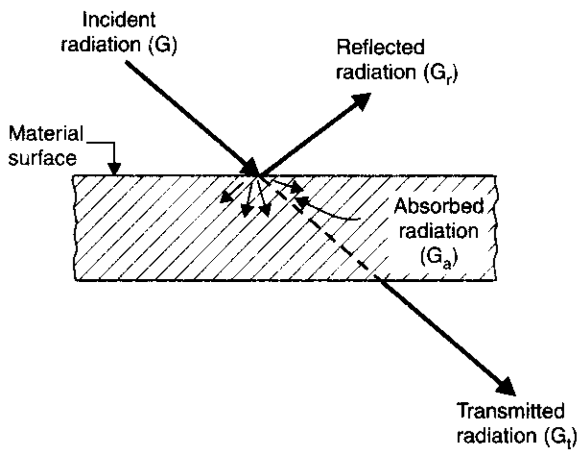


Fig. 15.46. Absorption, reflection and transmission of radiation.

By the conservation of energy principle,

$$G_a + G_r + G_t = G$$

Dividing both sides by G, we get

$$\frac{G_a}{G} + \frac{G_r}{G} + \frac{G_t}{G} = \frac{G}{G}$$

$$\alpha + \rho + \tau = 1 \qquad ...(15.67)$$

where, α = *Absorptivity* (or fraction of incident radiation absorbed),

ρ = *Reflectivity* (or fraction of incident radiation reflected), and

τ = *Transmittivity* (or fraction of incident radiation transmitted).

When the incident radiation is absorbed, it is converted into internal energy.

Black body. For perfectly absorbing body, $\alpha = 1$, $\rho = 0$, $\tau = 0$. Such a body is called a *'black body'* (*i.e., a black body is one which neither reflects nor transmits any part of the incident radiation but absorbs all of it*). In practice, a perfect black body ($\alpha = 1$) does not exist. However its concept is very important.

Opaque body. When no incident radiation is transmitted through the body, it is called an *'opaque body'*.

For the opaque body $\tau = 0$, and eqn. (15.67) reduces to

$$\alpha + \rho = 1 \qquad ...(15.68)$$

Solids generally do not transmit unless the material is of very thin section. Metals absorb radiation within a fraction of a micrometre, and insulators within a fraction of millimetre. Glasses and liquids are, therefore, generally considered as opaque.

White body. If all the incident radiation falling on the body are reflected, it is called a *'white body'*.

For a white body, $\rho = 1$, $\alpha = 0$ and $\tau = 0$.

Gases such as hydrogen, oxygen and nitrogen (and their mixture such as air) have a transmissivity of practically *unity*.

Reflections are of two types : Refer Fig. 15.47.

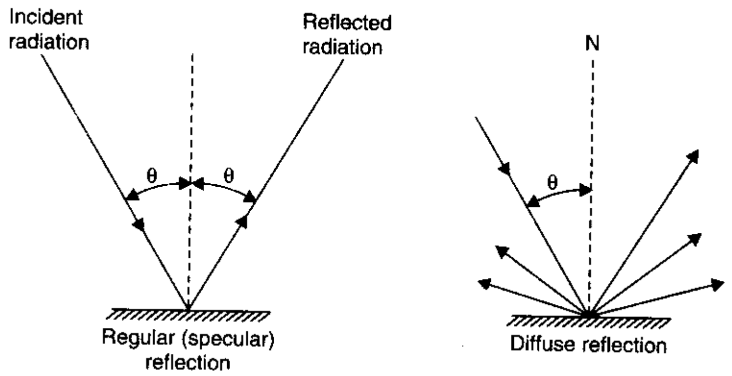


Fig. 15.47. Regular and diffuse reflections.

1. Regular (specular) reflection 2. Diffuse reflection.

Regular reflection implies that angle between the reflected beam and the normal to the surface *equals* the angle made by the incident radiation with the same normal. Reflection from highly polished and smooth surfaces approaches specular characteristics.

In a *diffused reflection,* the incident beam is reflected in *all directions*. Most of the engineering materials have rough surfaces, and these rough surfaces give diffused reflections.

Gray body. If the radiative properties, α, ρ τ of a body are assumed to be uniform over the entire wavelength spectrum, then such a body is called *gray body*. A *gray body* is also defined as one *whose absorptivity of a surface does not vary with temperature and wavelength of the incident radiation* [$\alpha = (\alpha)_\lambda$ = constant.]

A *coloured body* is one whose absorptivity of a surface *varies* with the wavelength of radiation [$\alpha \neq (\alpha)_\lambda$].

15.5.4. Concept of a Black Body

A black body is an object *that absorbs all the radiant energy reaching its surface* (for a black body $\alpha = 1$, $\rho = 0$, $\tau = 0$). No actual body is perfectly black ; the concept of a black body is an idealization with which the radiation characteristics of real bodies can be conveniently compared.

A *black body* has the following *properties* :

(*i*) It absorbs all the incident radiation falling on it and does not transmit or reflect regardless of wavelength and direction.

(*ii*) It emits maximum amount of thermal radiations at all wavelengths at any specified temperature.

(*iii*) It is a *diffuse emitter* (*i.e.,* the radiation emitted by a black body is independent of direction).

Consider a hollow enclosure with a very small hole for the passage of incident radiation as shown in Fig. 15.48. Incident radiant energy passes through the small opening ; some of this energy is absorbed by the inside surface and some is reflected. However, most of this energy is absorbed on a second incidence. Again, a small fraction is reflected. After a number of such reflections the amount unabsorbed is exceedingly small and very little of the original incident energy is reflected back out of the opening. A small hole leading into a cavity (Hohlraum) thus acts very nearly as a black body because all the radiant energy entering through it gets absorbed.

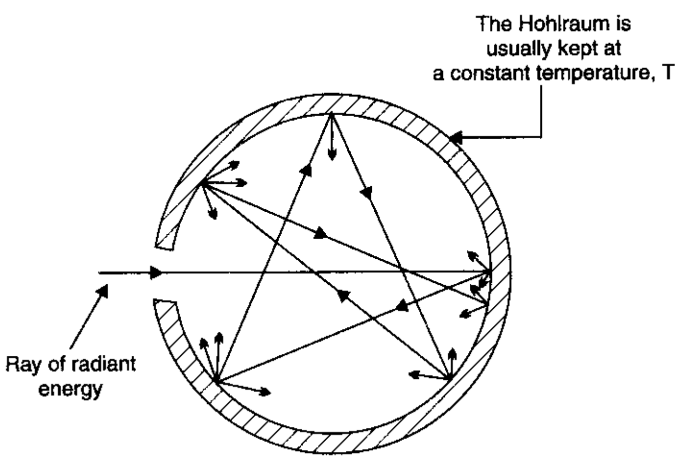


Fig. 15.48. Concept of a black body.

Isothermal furnaces, with small apertures, approximate a black body and are frequently used to *calibrate* heat flux gauges, thermometers and other radiometric devices.

15.5.5. The Stefan-Boltzmann Law

The law states that *the emissive power of a black body is directly proportional to the fourth power of its absolute temperature.*

i.e., $$E_b = \sigma T^4$$

where, E_b = Emissive power of a black body, and ...(15.69)

σ = Stefan-Boltzmann constant

$= 5.67 \times 10^{-8}$ W/m^2 K^4.

Equation (15.69) can be rewritten as :

$$E_b = 5.67 \left(\frac{T}{100}\right)^4 \quad ...(15.70)$$

15.5.6. Kirchhoff's Law

The law states that *at any temperature the ratio of total emissive power E to the total absorptivity α is a constant for all substances which are in thermal equilibrium with their environment.*

Let us consider a large radiating body of surface area A which encloses a small body (1) of surface area A_1 (as shown in Fig. 15.49). Let the energy fall on the unit surface of the body at the rate E_b. Of this energy, generally, a fraction α, will be absorbed by the small body. Thus this energy absorbed by the small body (1) is $\alpha_1 A_1 E_b$, in which α_1 is the absorptivity of the body. When thermal equilibrium is attained, the *energy absorbed* by the body (1) must be equal to the *energy emitted*, say, E_1 per unit surface. Thus, at equilibrium, we may write

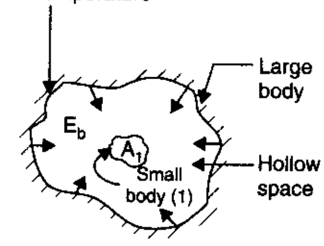


Fig. 15.49. Derivation of Kirchhoff's law.

$$A_1E_1 = \alpha_1 A_1 E_b \quad ...(15.71)$$

Now we remove body (1) and replace it by body (2) having absorptivity α_2. The radiative energy impinging on the surface of this body is again E_b. In this case, we may write

$$A_2E_2 = \alpha_2 A_2 E_b \quad ...(15.72)$$

By considering generality of bodies, we obtain

$$E_b = \frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E}{\alpha} \quad ...(15.73)$$

Also, as per definition of emissivity ε, we have

$$\varepsilon = \frac{E}{E_b}$$

or

$$E_b = \frac{E}{\varepsilon} \quad ...(15.74)$$

By comparing eqns. (15.73) and (15.74), we obtain

$$\varepsilon = \alpha \quad ...(15.75)$$

(α is always *smaller* than 1. Therefore, the emissive power E is always *smaller* than the emissive power of a black body at equal temperature).

Thus, kirchhoff's law also states that *the emissivity of a body is equal to its absorptivity when the body remains in thermal equilibrium with its surroundings.*

15.5.7. Planck's Law

In 1900 Max Planck showed by quantum arguments that the spectral distribution of the radiation intensity of a black body is given by

$$(E_\lambda)_b = \frac{2\pi c^2 h \lambda^{-5}}{\exp\left(\frac{ch}{\lambda k T}\right) - 1} \quad ...(\text{Planck's law}) \quad ...(15.76)$$

where, $(E_\lambda)_b$ = Monochromatic (single wavelength) emissive power of a black body,

c = Velocity of light in vacuum, $2.998 \times 10^8 \simeq 3 \times 10^8$ m/s,

h = Planck's constant = 6.625×10^{-34} js,

λ = Wavelength, μm,

k = Boltzmann constant = 1.3805×10^{-23} J/K, and

T = Absolute temperature, K.

Hence the unit of $(E_\lambda)_b$ is W/m^2. μm

Quite often the Planck's law is written as

$$(E_\lambda)_b = \frac{C_1\lambda^{-5}}{\exp\left[\frac{C_2}{\lambda T}\right] - 1} \qquad \text{...(15.77)}$$

where, $C_1 = 2\pi c^2 h = 3.742 \times 10^8$ W.μm^4/m^2 ;

$$C_2 = \frac{ch}{k} = 1.4388 \times 10^4 \text{ μmK}$$

Equation (15.76) is of great importance as it provides quantitative results for the radiation from a black body.

The quantity $(E_\lambda)_b$, *monochromatic emissive power*, is defined as the *energy emitted by the black surface in all directions at a given wavelength λ per unit wavelength interval around λ* ; that is, the rate of energy emission in the interval $d\lambda$ is equal to $(E_\lambda)_b\ d\lambda$. The total emissive power and monochromatic emissive power are related by the equation

$$E_b = \int_0^\infty (E_\lambda)_b\ d\lambda \qquad \text{...(15.78)}$$

A plot of $(E_\lambda)_b$ as a function of temperature and wavelength is given in Fig. 15.50.

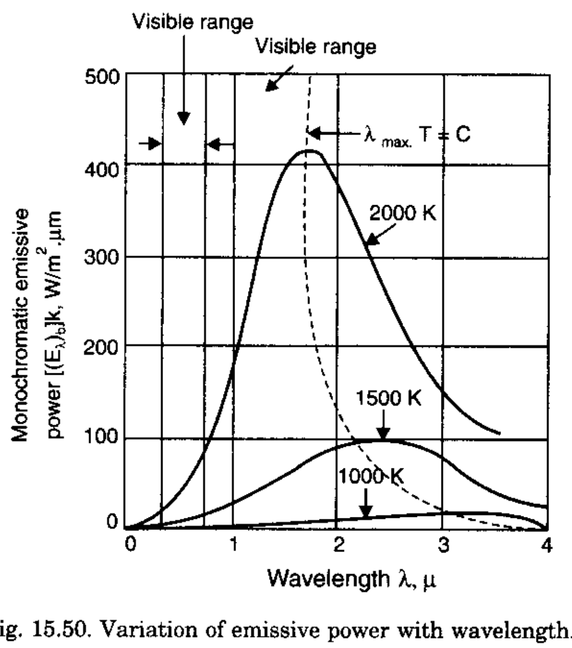


Fig. 15.50. Variation of emissive power with wavelength.

The plot shows that the following distinct characteristics of black body radiations :

1. *The energy emitted at all wavelengths increases with rise in temperature.*
2. *The peak spectral emissive power shifts towards a smaller wavelength at higher temperatures.* This shift signifies that at elevated temperature, much of the energy is emitted in a narrow band ranging on both sides of wavelength at which the monochromatic power is maximum.
3. The *area* under the monochromatic emissive power versus wavelength, at any temperature, gives the *rate of radiant energy emitted* within the wavelength interval $d\lambda$. Thus,

$$dE_b = (E_\lambda)_b \, d\lambda$$

or

$$E_b = \int_{\lambda=0}^{\lambda=\infty} (E_\lambda)_b \, d\lambda \qquad \text{... over the entire range of length.}$$

The integral represents the total emissive power per unit area radiated from a black body.

15.5.8. Wien's Displacement Law

In 1893 Wien established a relationship between the temperature of a black body and the wavelength at which the maximum value of monochromatic emissive power occurs. A peak monochromatic emissive power occurs at a particular wavelength. Wien's displacement law states that the *product of λ_{max} and T is constant, i.e.,*

$$\lambda_{max} T = \text{constant} \qquad \text{...(15.79)}$$

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1}$$

$(E_\lambda)_b$ becomes maximum (if T remains constant) when

$$\frac{d(E_\lambda)_b}{d\lambda} = 0$$

i.e.,

$$\frac{d(E_\lambda)_b}{d\lambda} = \frac{d}{d\lambda}\left[\frac{C_1 \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1}\right] = 0$$

or

$$\frac{\left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right](-5\,C_1\,\lambda^{-6}) - C_1\,\lambda^{-5}\left\{\exp\left(\frac{C_2}{\lambda T}\right)\frac{C_2}{T}\left(-\frac{1}{\lambda^2}\right)\right\}}{\left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]^2} = 0$$

or

$$-5\,C_1\,\lambda^{-6}\exp\left(\frac{C_2}{\lambda T}\right) + 5\,C_1\,\lambda^{-6} + C_1\,C_2\,\lambda^{-5}\,\frac{1}{\lambda^2 T}\exp\left(\frac{C_2}{\lambda T}\right) = 0$$

Dividing both side by $5C_1\,\lambda^{-6}$, we get

$$-\exp\left(\frac{C_2}{\lambda T}\right) + 1 + \frac{1}{5}\,C_2\,\frac{1}{\lambda T}\exp\left(\frac{C_2}{\lambda T}\right) = 0$$

Solving this equation by trial and error method, we get

$$\frac{C_2}{\lambda T} = \frac{C_2}{\lambda_{max} T} = 4.965$$

$\therefore$

$$\lambda_{max}\,T = \frac{C_2}{4.965} = \frac{1.439 \times 10^4}{4.965}\ \mu\text{mK} = 2898\ \mu\text{mK}\ (\simeq 2900\ \mu\text{mK})$$

i.e.,

$$\lambda_{max}\,T = 2898\ \mu\text{mK} \qquad \text{...(15.80)}$$

This law holds true for more *real substances* ; there is however some deviation in the case of a metallic conductor where the product ($\lambda_{max}.T$) is found to vary with absolute temperature. It is used in *predicting a very high temperature through measurement of wavelength.*

A *combination of Planck's law and Wien's displacement law yields the condition for the maximum monochromatic emissive power for a black body.*

$$(E_{\lambda b})_{max} = \frac{C_1 (\lambda_{max})^{-5}}{\exp\left[\dfrac{C_2}{\lambda_{max}\, T}\right] - 1} = \frac{0.374 \times 10^{-15} \left(\dfrac{2.898 \times 10^{-3}}{T}\right)^{-5}}{\exp\left[\dfrac{1.4388 \times 10^{-2}}{2.898 \times 10^{-3}}\right] - 1}$$

or $$(E_{\lambda b})_{max} = 1.285 \times 10^{-5}\, T^5 \text{ W/m}^2 \text{ per metre wavelength} \qquad ...(15.81)$$

15.5.9. Intensity of Radiation and Lambert's Consine Law

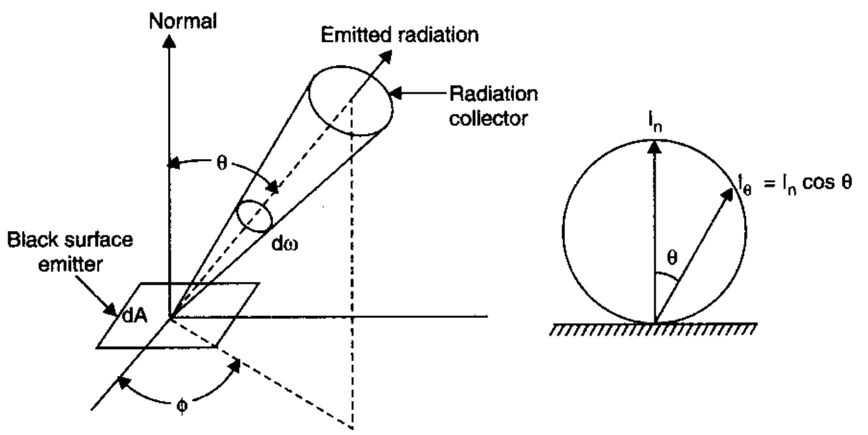
15.5.9.1. Intensity of Radiation

When a surface element emits radiation, all of it will be intercepted by a hemispherical surface placed over the element. The **intensity of radiation** (I) is defined as the *rate of energy leaving a surface in a given direction per unit solid angle per unit area of the emitting surface normal to the mean direction in space.* A **solid angle** is defined as a *portion of the space inside a sphere enclosed by a conical surface with the vertex of the cone at the centre of the sphere.* It is measured by the ratio of the spherical surface enclosed by the cone to the square of the radius of the sphere ; it unit is steradian (sr). The solid angle subtended by the complete hemisphere is given by : $\dfrac{2\pi r^2}{r^2} = 2\pi$.

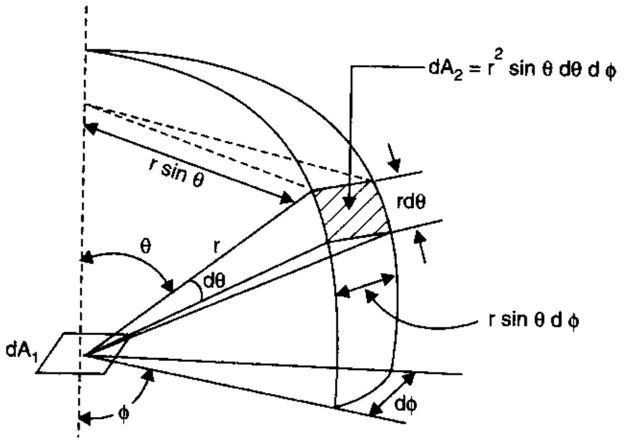
Fig. 15.51 (*a*) shows a small black surface of area dA (emitter) emitting radiation in different directions. A black body radiation collector through which the radiation pass is located at an angular position characterised by *zenith angle* θ towards the surface normal and angle ϕ of a spherical coordinate system. Further the collector subtends a solid angle $d\omega$ when viewed from a point on the emitter.

Let us now consider radiation from the elementary area dA_1 at the centre of a sphere as shown in Fig. 15.51. Suppose this radiation is absorbed by a second elemental area dA_2, a portion of the hemispherical surface.

The projected area of dA_1 on a plane perpendicular to the line joining dA_1 and $dA_2 = dA_1 \cos\theta$.



(*a*) Special distribution of radiations emitted from a surface.



(*b*) Illustration for evaluating area dA_2

Fig. 15.51. Radiation from an elementary surface.

The solid angle subtended by $dA_2 = \dfrac{dA_2}{r^2}$

$\therefore$ The intensity of radiation,

$$I = \frac{dQ_{1-2}}{dA_1 \cos\theta \times \dfrac{dA_2}{r^2}} \qquad \text{...(15.82)}$$

where dQ_{1-2} is the rate of radiation heat transfer from dA_1 to dA_2.

It is evident from Fig. 15.51 (*b*) that,

$$dA_2 = r\, d\theta\, (r \sin\theta\, d\phi)$$

or

$$dA_2 = r^2 \sin\theta . d\theta . d\phi \qquad \text{...(15.83)}$$

From eqns. (15.82) and (15.83), we obtain

$$dQ_{1-2} = I\, dA_1 \,.\, \sin\theta \,.\, \cos\theta \,.\, d\theta \,.\, d\phi$$

The total radiation through the hemisphere is given by

$$Q = I\, dA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{\phi=0}^{\phi=2\pi} \sin\theta \cos\theta\, d\theta\, d\phi$$

$$= 2\pi I\, dA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \sin\theta \cos\theta\, d\theta$$

$$= \pi I\, dA_1 \int_{\theta=0}^{\frac{\pi}{2}} 2 \sin\theta \cos\theta\, d\theta$$

$$= \pi I\, dA_1 \int_{\theta=0}^{\frac{\pi}{2}} \sin 2\theta\, d\theta$$

or

$$Q = \pi I\, dA_1 \qquad \text{...(15.84)}$$

Also

$$Q = E . dA_1$$

$\therefore$

$$E\, dA_1 = \pi I\, dA_1$$

or

$$E = \pi I$$

i.e., The *total emissive power of a diffuse surface is equal to* π *times its intensity of radiation.*

15.5.9.2. Lambert's Cosine Law

The law states that the *total emissive power E_θ from a radiating plane surface in any direction is directly proportional to the cosine of the angle of emission.* The angle of emission θ is the angle subtended by the normal to the radiating surface and the direction vector of emission of the receiving surface. If E_n be the total emissive power of the radiating surface in the direction of its normal, then

$$E_\theta = E_n \cos\theta \qquad \text{...(15.85)}$$

The above equation is true only for diffuse radiation surface. The radiation emanating from a point on a surface is termed diffused if the intensity, I is constant. This law is also known as *Lambert's law of diffuse radiation.*

Example 15.23. *The effective temperature of a body having an area of 0.12 m² is 527°C. Calculate the following :*

(*i*) *The total rate of energy emission.*

(*ii*) *The intensity of normal radiation, and*

(*iii*) *The wavelength of maximum monochromatic emissive power.*

Solution. *Given* : $A = 0.12 \text{ m}^2$; $T = 527 + 273 = 800$ K

(*i*) **The total rate of energy emission, E_b :**

$$E_b = \sigma A T^4 \text{ W (watts)} \qquad \text{...[Eqn. 15.64 (a)]}$$

$$= 5.67 \times 10^{-8} \times 0.12 \times (800)^4 = 5.67 \times 0.12 \times \left(\frac{800}{100}\right)^4 = \mathbf{2786.9\ W.} \quad \textbf{(Ans.)}$$

(*ii*) **The intensity of normal radiation, I_{bn} :**

$$I_{bn} = \frac{E_b}{\pi}, \text{ where } E_b \text{ is in W/m}^2\text{ K}^4$$

$$= \frac{\sigma T^4}{\pi} = \frac{5.67 \times \left(\frac{800}{100}\right)^4}{\pi} = \mathbf{7392.5\ W/m^2.sr.} \quad \textbf{(Ans.)}$$

(*iii*) **The wavelength of maximum monochromatic emissive power, λ_{max} :**

From Wien's displacement law,

$$\lambda_{max}.T = 2898\ \mu\text{m K} \qquad \text{...[Eqn. 15.80]}$$

or

$$\lambda_{max} = \frac{2898}{T} = \frac{2898}{800} = \mathbf{3.622\ \mu m.} \quad \textbf{(Ans.)}$$

Example 15.24. *Assuming the sun to be a black body emitting radiation with maximum intensity at λ = 0.49 μm, calculate the following :*

(*i*) *The surface temperature of the sun, and*

(*ii*) *The heat flux at surface of the sun.*

Solution. Given : $\lambda_{max} = 0.49\ \mu$m

(*i*) **The surface temperature of the sun, T :**

According to Wien's displacement law,

$$\lambda_{max} . T = 2898\ \mu\text{mK}$$

$$\therefore \quad T = \frac{2898}{\lambda_{max}} = \frac{2898}{0.48} = \mathbf{5914\ K.} \quad \textbf{(Ans.)}$$

(*ii*) **The heat flux at the surface of the sun, $(E)_{sun}$:**

$$(E)_{sun} = \sigma T^4 = 5.67 \times 10^{-8}\ T^4 = 5.67 \left(\frac{T}{100}\right)^4$$

$$= 5.67 \times \left(\frac{5914}{100}\right)^4 = \mathbf{6.936 \times 10^7\ W/m^2}$$

Example 15.25. *Calculate the following for an industrial furnance in the form of a black body and emitting radiation of 2500°C :*

(*i*) *Monochromatic emissive power at 1.2 μm length,*

(*ii*) *Wavelength at which the emission is maximum,*

(*iii*) *Maximum emissive power,*

(*iv*) *Total emissive power, and*

(*v*) *Total emissive power of the furnance if it is assumed as a real surface with emissivity equal to 0.9.*

Solution. *Given :* $T = 2500 + 273 = 2773$ K ; $\lambda = 12$ μm, $\varepsilon = 0.9$

(*i*) **Monochromatic emissive power at 1.2 μm length, $(E_\lambda)_b$:**

According to Planck's law, $$(E_\lambda)_b = \frac{C_1\lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1}$$ ...[Eqn. (15.77)]

where, $C_1 = 3.742 \times 10^8$ W. μm^4/m^2 = 0.3742×10^{-15} W.m^4/m^2 and

$C_2 = 1.4388 \times 10^{-2}$ mK

Substituting the values, we get

$$(E_\lambda)_b = \frac{0.3742 \times 10^{-15} \times (1.2 \times 10^{-6})^{-5}}{\exp\left(\frac{1.4388 \times 10^{-4}}{1.2 \times 10^{-6} \times 2773}\right) - 1} = \frac{1.5 \times 10^{14}}{74.48}$$

$$= \mathbf{2.014 \times 10^{12}\ W/m^2.}\quad \textbf{(Ans.)}$$

(*ii*) **Wavelength at which the emission is maximum, λ_{max} :**

According to Wien's displacement law,

$$\lambda_{max} = \frac{2898}{T} = \frac{2898}{2773} = \mathbf{1.045\ \mu m.}\quad \textbf{(Ans.)}$$

(*iii*) **Maximum emissive power, $(E_{\lambda b})_{max}$:**

$$(E_{\lambda b})_{max} = 1.285 \times 10^{-5}\, T^5\ \text{W/m}^2 \text{ per metre length}$$ ...[Eqn. (15.81)]

$$= 1.285 \times 10^{-5} \times (2773)^5 = \mathbf{2.1 \times 10^{12}\ W/m^2\ per\ metre\ length.}\quad \textbf{(Ans.)}$$

[**Note.** At high temperature the difference between $(E_\lambda)_b$ and $(E_{\lambda b})_{max}$ is very small].

(*iv*) **Total emissive power, E_b :**

$$E_b = \sigma T^4 = 5.67 \times 10^{-8}\,(2273)^4 = 5.67\left(\frac{2773}{100}\right)^4 = \mathbf{3.352 \times 10^6\ W/m^2}$$

(*v*) **Total emissive power, E with emisivity (ε) = 0.9**

$$E = \varepsilon\,\sigma T^4 = 0.9 \times 5.67\left(\frac{2773}{100}\right)^4 = \mathbf{3.017 \times 10^6\ W/m^2.}\quad \textbf{(Ans.)}$$

15.5.10. Radiation Exchange Between Black Bodies Separated by a Non-absorbing Medium

Refer Fig. 15.52. Let us consider heat exchange between elementary areas dA_1 and dA_2 of two black radiating bodies, separated by a non-absorbing medium, and having areas A_1 and A_2 and temperatures T_1 and T_2 respectively. The elemenary areas are at a distance r apart and the normals to these areas make angles θ_1 and θ_2 with the line joining them. Each elemental area subtends a solid angle at the centre of the other. Let $d\omega_1$ be subtended at dA_1 by dA_2 and $d\omega_2$ subtended at dA_2 by dA_1. Then

$$d\omega_1 = \frac{dA_2 \cos \theta_2}{r^2}, \text{ and } d\omega_2 = \frac{dA_1 \cos \theta_1}{r^2} \quad ...(15.86)$$

The energy leaving dA_1 in the direction given by the angle per unit solid angle = $I_{b_1}\, dA_1 \cos \theta_1$.

where, I_b = Black body intensity, and

$dA_1 \cos \theta_1$ = Projection of dA_1 on the line between the centres.

The rate of radiant energy leaving dA_1 and striking on dA_2 is given by

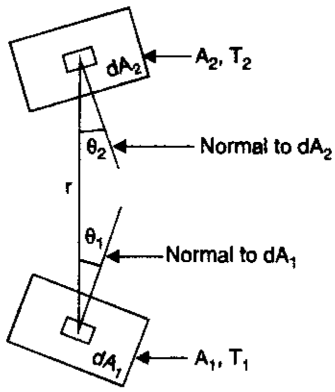


Fig. 15.52. Radiation heat exchange between two black surfaces.

$$dQ_{1-2} = I_{b_1}\, dA_1 \cos \theta_1 .\, d\omega_1$$

$$= \frac{I_{b_1} \cos \theta_1 \cos \theta_2\, dA_1\, dA_2}{r^2} \quad ...(15.87)$$

This energy is absorbed by the elementary area dA_2, since both the surfaces are black. The quantity of energy radiated by dA_2 and absorbed by dA_1 is given by

$$dQ_{2-1} = \frac{I_{b_2} \cos \theta_2 \cos \theta_1\, dA_2\, dA_1}{r^2} \quad ...(15.88)$$

The net rate of transfer of energy between dA_1 and dA_2 is

$$dQ_{12} = dQ_{1-2} - dQ_{2-1}$$

$$= \frac{dA_1\, dA_2 \cos \theta_1 \cos \theta_2}{r^2} \left(I_{b_1} - I_{b_2}\right)$$

But $$I_{b_1} = \frac{E_{b_1}}{\pi} \text{ and } I_{b_2} = \frac{E_{b_2}}{\pi} \quad ...[\text{Eqn. } (15.88)]$$

$\therefore$ $$dQ_{12} = \frac{dA_1\, dA_2 \cos \theta_1 \cos \theta_2}{\pi r^2} (E_{b_1} - E_{b_2}) \quad ...(15.89)$$

or $$dQ_{12} = \frac{\sigma dA_1\, dA_2 \cos \theta_1 \cos \theta_2}{\pi r^2} (T_1^2 - T_2^4) \quad ...(15.90)$$

The rate of total net heat transfer for the total areas A_1 and A_2 is given by

$$Q_{12} = \int dQ_{12} = \sigma (T_1^2 - T_2^4) \int_{A_1}\int_{A_2} \frac{\cos \theta_1 \cos \theta_2\, dA_1\, dA_2}{r^2} \quad ...(15.91)$$

The rate of radiant energy emitted by A_1 that falls on A_2, from eqn. (15.87), is given by

$$Q_{1-2} = I_{b_1} \int_{A_1}\int_{A_2} \frac{\cos \theta_1 \cos \theta_2\, dA_1\, dA_2}{r^2}$$

$$Q_{1-2} = \sigma T_1^4 \int_{A_1}\int_{A_2} \frac{\cos \theta_1 \cos \theta_2\, dA_1\, dA_2}{\pi r^2} \quad ...(15.92)$$

The rate of total energy radiated by A_1 is given by,

$$Q_1 = A_1\, \sigma\, T_1^4$$

Hence the fraction of the rate of energy leaving area A_1 and impinging on area A_2 is given by

$$\frac{Q_{1-2}}{Q_1} = \frac{1}{A_1} \int_{A_1}\int_{A_2} \frac{\cos \theta_1 \cos \theta_2\, dA_1\, dA_2}{\pi r^2} \quad ...(15.93)$$

or $$\frac{Q_{1-2}}{Q_1} = F_{1-2} \quad ...[15.93\ (a)]$$

F_{1-2} is known as '**configuration factor**' or '**surface factor**' or '**view factor**' between the two radiating surfaces and is a function of geometry only.

Thus, the ***shape factor*** may be defined as "*The fraction of radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening reflections.*"

Further, $$Q_{1-2} = F_{1-2} A_1 \sigma T_1^4 \quad ...(15.94)$$

Similarly, the rate of radiant energy by A_2 that falls on A_1, from eqn. (15.88), is given by

$$Q_{2-1} = \sigma T_2^4 \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{\pi r^2}$$

The rate of total energy radiated by A_2 is given by

$$Q_2 = A_2 \sigma T_2^4$$

Hence the fraction of the rate of energy leaving area A_2 and impinging on area A_1 is given by

$$\frac{Q_{2-1}}{Q_2} = \frac{1}{A_2} \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{\pi r^2} \quad ...(15.95)$$

or $$\frac{Q_{2-1}}{Q_2} = F_{2-1}$$

F_{2-1} is the shape factor of A_2 with respect to A_1.

$$Q_{2-1} = F_{2-1} A_2 \sigma T_2^4 \quad ...(15.96)$$

From eqns. (15.93) and (15.95), we get

$$A_1 F_{1-2} = A_2 F_{2-1} \quad ...(15.97)$$

The above result is known as **reciprocity theorem.** It indicates that the net radiant interchange may be evaluated by computing one way configuration factor from either surface to the other. Thus the net rate of heat transfer between two surfaces A_1 and A_2 is given by

$$Q_{12} = A_1 F_{1-2} \sigma (T_1^4 - T_2^4)$$
$$= A_2 F_{2-1} \sigma (T_1^4 - T_2^4) \quad ...(15.98)$$

It may be noted that eqn. (15.98) is applicable to *black surfaces only and must not be used* for surfaces having emissivities very different from unity.

Example 15.26. *A body at 1000°C in black surroundings at 500°C has an emissivity of 0.42 at 1000°C and an emissivity of 0.72 at 500°C. Calculate the rate of heat loss by radiation per m².*

(i) When the body is assumed to be grey with ε = 0.42.

(ii) When the body is not grey.

Assume that the absorptivity is independent of the surface temperature.

Solution. (*i*) **When the body is grey with ε = 0.42 :**

$$T_1 = 1000 + 273 = 1273 \text{ K}$$
$$T_2 = 500 + 273 = 773 \text{ K}$$
$$\varepsilon \text{ at } 1000°\text{C} = 0.42$$
$$\varepsilon \text{ at } 500°\text{C} = 0.72$$
$$\sigma = 5.67 \times 10^{-8}$$

Heat loss per m² by radiation,

$$q = \varepsilon\sigma (T_1^4 - T_2^4)$$
$$= 0.42 \times 5.67 \times 10^{-8} [(1273)^4 - 773)^4] = 54893 \text{ W}$$

i.e., *Heat loss per m² by radiation* = **54.893 kW. (Ans.)**

(*ii*) **When the body is not grey :**

Absorptivity when source is at 500°C = Emissivity when body is at 500°C

i.e., absorptivity, $\alpha = 0.72$

Then, energy emitted $= \varepsilon\sigma\, T_1^4 = 0.42 \times 5.67 \times 10^{-8} \times (1273)^4$

and, Energy absorbed $= \alpha\sigma T_2^4 = 0.72 \times 5.67 \times 10^{-8} \times (773)^4$

i.e., q = Energy emitted – Energy absorbed

$= 0.42 \times 5.67 \times 10^{-8} \times (1238)^4 - 0.72 \times 5.67 \times 10^{-8} \times (773)^4$

$= 62538 - 14576 = 47962$ W

i.e., Heat loss per m^2 by radiation = **47.962 kW. (Ans.)**

Example 15.27. *A long steel rod, 22 mm in diameter, is to be heated from 420°C to 540°C. It is placed concentrically in a long cylindrical furnace which has an inside diameter of 180 mm. The inner surface of the furnace is at a temperature of 1100°C, and has an emissivity of 0.82. If the surface of the rod has an emissivity of 0.62, find the time required for the heating operation.*

Take for steel : c = 0.67 kJ/kg K, ρ= 7845 kg/m³.

Solution. Refer Fig. 15.53.

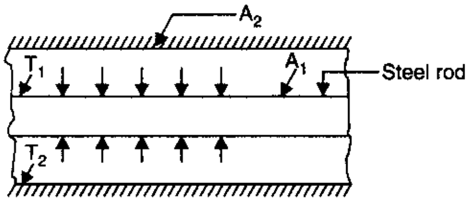


Fig. 15.53

Diameter of the steel rod

= 22 mm = 0.022 m

Inside diameter of the furnace

= 180 mm = 0.18 m

Emissivity $\varepsilon_1 = 0.62$

Emissivity $\varepsilon_2 = 0.82$

Specific heat of steel, $c = 0.67$ kJ/kg K

Density of steel, $\rho = 7845$ kg/m³

$T_1 = 420 + 273 = 693$ K ..... 1st case

and $= 540 + 273 = 813$ K ..... 2nd case

$T_2 = 1100 + 273 = 1373$ K

The surface area of the rod, $A_1 = \pi \times 0.022 \times l$ m²

The surface area of the furnace, $A_2 = \pi \times 0.18 \times l$ m²

Time required for the heating operation, t_h :

Initial rate of heat absorption by radiation, when the rod is at 420°C or 693 K

$$Q_i = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{\pi \times 0.022 \times 1 \times 5.67 \times 10^{-8}\,(693^4 - 1373^4)}{\frac{1}{0.62} + \left(\frac{\pi \times 0.022 \times l}{\pi \times 0.18 \times l}\right)\left[\frac{1}{0.82} - 1\right]}$$

$$= \frac{-13022.5}{1.64} = -7940.5 \text{ W/m} \qquad [\because\ l = 1\,m\ldots\text{assumed}]$$

Rate of heat absorption at the end of the heating process, when the rod is at 540°C or 813 K

$$Q_e = \frac{A_2\,\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{\pi \times 0.022 \times 1 \times 5.67 \times 10^{-8} (813^4 - 1373^4)}{\frac{1}{0.62} + \frac{\pi \times 0.022 \times l}{\pi \times 0.18 \times l}\left[\frac{1}{0.82} - 1\right]}$$

$$= \frac{-12214.3}{1.64} = -7447.7 \text{ W/m}$$

∴ Average rate of heat absorption during the heating process

$$Q_{av.} = \frac{7940.5 + 7447.7}{2} = 7694.1 \text{ W/m}$$

Time required for heating, t_h is obtained from the equation

$$mc_p \, \Delta T = Q_{av.} \times t_h$$

∴ $$t_h = \frac{[\pi/4 \times (0.022)^2 \times 1 \times 7845] \times 0.67 \times (540 - 420) \times 1000}{7694.1}$$

$$= \mathbf{31.16 \text{ s.}} \quad \textbf{(Ans.)}$$

Example 15.28. *Calculate the heat transfer rate per m² area by radiation between the surfaces of two long cylinders having radii 100 mm and 50 mm respectively. The smaller cylinder being in the larger cylinder. The axes of the cylinders are parallel to each other and separated by a distance of 20 mm. The surfaces of inner and outer cylinders are maintained at 127°C and 27°C respectively. The emissivity of both the surfaces is 0.5.*

Assume the medium between the two cylinders is non-absorbing. **(P.U.)**

Solution. *Given* : $r_1 = 50$ mm = 0.05 m ; $r_2 = 100$ mm = 0.1 m, $T_1 = 127 + 273 = 400$ K,

$T_2 = 27 + 273 = 300$ K, $\varepsilon_1 = \varepsilon_2 = 0.5$

The heat transfer between two concentric or eccentric cylinders is given by

$$(Q_{12})_{net} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + \frac{1}{F_{1-2}} + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{A_1}{A_2}}$$

Here $F_{1-2} = 1$ and $\frac{A_1}{A_2} = \frac{2\pi r_1 L}{2\pi r_2 L} = \frac{r_1}{r_2}$

Substituting the values, we have

$$(Q_{12})_{net} = \frac{1 \times 5.67\left[\left(\frac{400}{100}\right)^4 - \left(\frac{300}{100}\right)^4\right]}{\left(\frac{1-0.5}{0.5}\right) + 1\left(\frac{1-0.5}{0.5}\right) \times \frac{0.05}{0.1}} = \frac{992.25}{2.5} = \mathbf{396.9 \text{ W/m}^2}. \quad \textbf{(Ans.)}$$

Example 15.29. *Three thin walled infinitely long hollow cylinders of radii 5 cm, 10 cm and 15 cm are arranged concentrically as shown in Fig. 15.54. T_1 = 1000 K and T_3 = 300 K.*

Assuming $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.05$ and vacuum in the spaces between the cylinders, calculate the steady state temperature of cylinder surface 2 and heat flow per m² area of cylinder 1. **(P.U.)**

Solution. *Given* : $r_1 = 5$ cm ; $r_2 = 10$ cm ; $r_3 = 15$ cm ; $T_1 = 1000$ K ; $T_3 = 300$ K

$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.05.$

For steady state heat flow,

$$Q_{12} = Q_{23}$$

or

$$\frac{A_1 \sigma (T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + \frac{1}{F_{1-2}} + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{A_1}{A_2}} = \frac{A_2 \sigma (T_2^4 - T_3^4)}{\left(\frac{1-\varepsilon_2}{\varepsilon_2}\right) + \frac{1}{F_{2-3}} + \left(\frac{1-\varepsilon_3}{\varepsilon_3}\right)\frac{A_2}{A_3}}$$

In this case $F_{1-2} = F_{2-3} = 1$; and

$$\frac{A_1}{A_2} = \frac{r_1}{r_2} = \frac{5}{10} = 0.5$$

$$\frac{A_2}{A_3} = \frac{r_2}{r_3} = \frac{10}{15} = 0.67$$

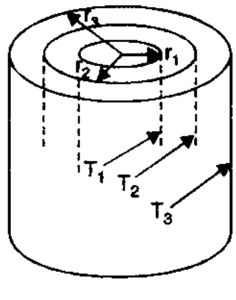


Fig. 15.54

$$\therefore \quad \frac{2\pi r_1 L\left[\left(\frac{1000}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{\left(\frac{1-0.05}{0.05}\right) + 1 + \left(\frac{1-0.05}{0.05}\right) \times 0.5}$$

$$\frac{2\pi r_2 L\left[\left(\frac{T_2}{100}\right)^4 - \left(\frac{300}{100}\right)^4\right]}{\left(\frac{1-0.05}{0.05}\right) + 1 + \left(\frac{1-0.05}{0.05}\right) \times 0.67}$$

or $$\frac{0.05(10000 - x^4)}{29.4} = \frac{0.1(x^4 - 81)}{32.73}$$

or $$(1000 - x^4) = \frac{29.5 \times 0.1}{32.73 \times 0.05}(x^4 - 81) = 1.8(x^4 - 81)$$

or $$2.8x^4 = 10000 - 145.8 = 9854.2$$

or $$x = \left(\frac{9854.2}{2.8}\right)^{1/4} = 7.7$$

or $$\frac{T_2}{100} = 7.7 \text{ or } T_2 = 770 \text{ K}$$

$\therefore$ Heat flow per m^2 area of cylinder 1,

$$Q_{12} = \frac{A_1 \sigma(T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + 1 + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{A_1}{A_2}}$$

$$= \frac{1 \times 5.67\left[\left(\frac{1000}{100}\right)^4 - \left(\frac{770}{100}\right)^4\right]}{\left(\frac{1-0.05}{0.05}\right) + 1 + \left(\frac{1-0.05}{0.05}\right) \times 0.5}$$

$$= \frac{5.67 \times (10000 - 3515.3)}{29.5} = \mathbf{1246.4\ W.} \quad \textbf{(Ans.)}$$

Example 15.30. *Two concentric spheres 210 mm and 300 mm diameters with the space between them evacuated are to be used to store liquid air (– 153°C) in a room at 27°C. The surfaces of the spheres are flushed with aluminium (ε = 0.03) and latent heat of vaporization of liquid air is 209.35 kJ/kg. Calculate the rate of evaporation of liquid air.* **(M.U.)**

Solution. *Given* : $r_1 = \frac{210}{2} = 105$ mm = 0.105 m ; $r_2 = \frac{300}{2} = 150$ mm = 0.15 m ;

$T_1 = -153 + 273 = 120$ K ; $T_2 = 27 + 273 = 300$ K ; $\varepsilon_1 = \varepsilon_2 = 0.03$, $h_{fg} = 209.35$ kJ/kg.

Rate of evaporation of liquid air :

The heat flow from the inner sphere surface to outer sphere surface is given by,

$$Q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + \frac{1}{F_{1-2}} + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{A_1}{A_2}}$$

$$= \frac{4\pi r_1^2\sigma(T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + 1 + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{r_1^2}{r_2^2}}$$

$$= \frac{4\pi \times (0.105)^2 \times 5.67\left[\left(\frac{120}{100}\right)^4 - \left(\frac{300}{100}\right)^4\right]}{\left(\frac{1-0.03}{0.03}\right) + 1 + \left(\frac{1-0.03}{0.03}\right) \times \left(\frac{0.105}{0.15}\right)^2}$$

$$= \frac{0.7855\,(2.07 - 81)}{32.33 + 1 + 15.84} = \frac{-61.99}{49.17} = -1.26 \text{ W}$$

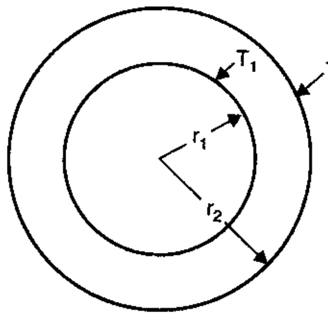


Fig. 15.55

– ve sign indicates that heat is gained by the surface 1, means, heat is flowing from outside surface to inside surface.

$\therefore$ The rate of evaporation $= \frac{1.26 \times 3600}{209.35 \times 1000}$ = **0.0217 kg/h. (Ans.)**

Example 15.31. *Liquid oxygen (boiling temperature = – 182°C) is to be stored in spherical container of 30 cm diameter. The system is insulated by an evacuated space between inner sphere and surrounding 45 cm inner diameter concentric sphere. For both spheres ε = 0.03 and temperature of the outer sphere is 30°C. Estimate the rate of heat flow by radiation to the oxygen in the container.*

Solution. *Given :* $T_1 = -182 + 273 = 91$ K, $T_2 = 30 + 273 = 303$ K, $\varepsilon_1 = \varepsilon_2 = 0.03$

$d_1 = 30$ cm $= 0.3$ m, $d_2 = 45$ cm $= 0.45$ m.

Rate of heat flow, Q_{12} :

The heat flow between the two concentric spheres by radiation is given by

$$Q_{12} = \frac{A_1\,\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{1-2}} + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{A_1}{A_2}}$$

For concentric spheres

$F_{1-2} = 1$

and $\frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{0.3}{0.45}\right)^2 = 0.4444$

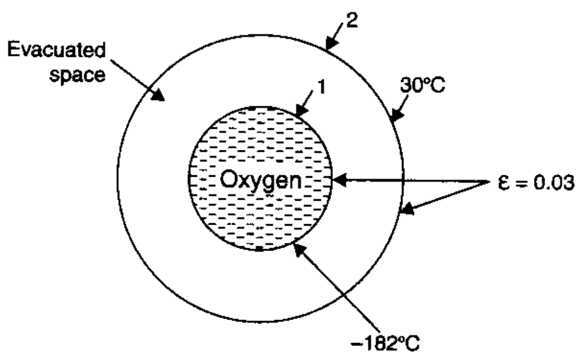


Fig. 15.56

$$A_1 = 4\pi\, r_1^2 = 4\pi \times \left(\frac{0.3}{2}\right)^2 = 0.283 \text{ m}^2$$

Now substituting the values in the above equation, we get

$$Q_{12} = \frac{0.283 \times 5.67\left[\left(\frac{91}{100}\right)^4 - \left(\frac{303}{100}\right)^4\right]}{\left(\frac{1-0.03}{0.03}\right) + 1 + \left(\frac{1-0.03}{0.03}\right) \times 0.4444}$$

$$= \frac{0.283 \times 5.67(0.686 - 84.289)}{32.33 + 1 + 14.37} = -\,\mathbf{2.81\ W}$$

– ve sign shows *heat flows from outside to inside.* **(Ans.)**

Example 15.32 (Radiation shield). *The large parallel planes with emissivities 0.3 and 0.8 exchange heat. Find the percentage reduction when a polished aluminium shield of emissivity 0.04 is placed between them. Use the method of electrical analogy.*

Solution. *Given :* $\varepsilon_1 = 0.3$; $\varepsilon_2 = 0.8$; $\varepsilon_3 = 0.04$

Consider all resistances (surface resistances and space resistances) per unit surface area.

For steady state heat flow,

$$\frac{E_{b_1} - E_{b_3}}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + 1 + \left(\frac{1-\varepsilon_3}{\varepsilon_3}\right)} = \frac{E_{b_3} - E_{b_2}}{\left(\frac{1-\varepsilon_3}{\varepsilon_3}\right) + 1 + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)}$$

[$\because$ $A_1 = A_2 = A_3 = 1\ \text{m}^2$ and $F_{1-3}, F_{3-2} = 1$]

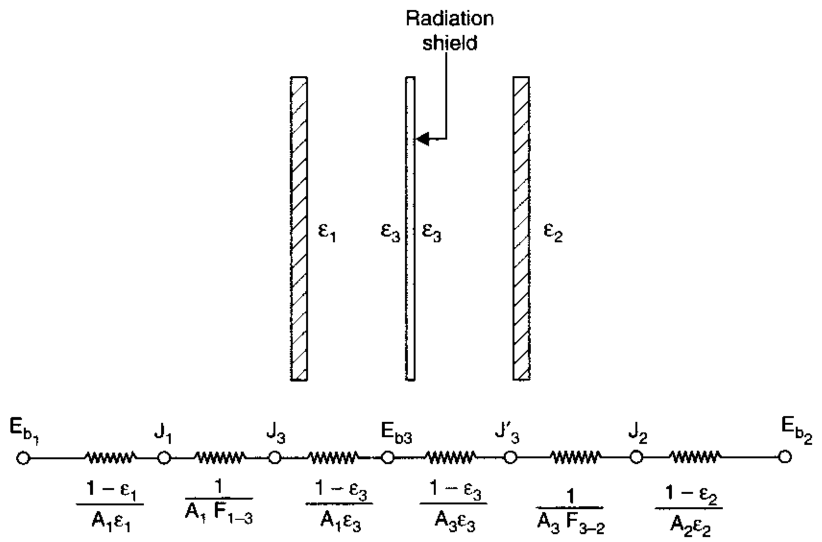


Fig. 15.57

or
$$\frac{\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\sigma(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

or
$$\frac{T_1^4 - T_3^4}{\frac{1}{0.3} + \frac{1}{0.04} - 1} = \frac{T_3^4 - T_2^4}{\frac{1}{0.04} + \frac{1}{0.8} - 1}$$

or $$\frac{T_1^4 - T_3^4}{27.33} = \frac{T_3^4 - T_2^4}{25.25}$$

or $$T_1^4 - T_3^4 = \frac{27.33}{25.25}(T_3^4 - T_2^4)$$

$$= 1.08\ (T_3^4 - T_2^4) = 1.08\ T_3^4 - 1.08\ T_2^4$$

or $$2.08\ T_3^4 = T_1^4 + 1.08\ T_2^4$$

or $$T_3^4 = \frac{1}{2.08}\ (T_1^4 + 1.08\ T_2^4) = 0.48\ (T_1^4 + 1.08\ T_2^4) \quad ...(i)$$

Q_{12} (heat flow **without shield**)

$$= \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{0.3} + \frac{1}{0.8} - 1} = \frac{\sigma(T_1^4 - T_2^4)}{3.58} \quad ...(ii)$$

Q_{13} (heat flow **with shield**)

$$= \frac{\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\sigma(T_1^4 - T_3^4)}{\frac{1}{0.3} + \frac{1}{0.4} - 1} = \frac{\sigma(T_1^4 - T_3^4)}{27.33} \quad ...(iii)$$

∴ **Percentage reduction** in heat flow due to shield

$$= \frac{Q_{12} - Q_{13}}{Q_{12}}$$

$$= 1 - \frac{Q_{13}}{Q_{12}} = 1 - \frac{\sigma(T_1^4 - T_3^4)/27.33}{\sigma(T_1^4 - T_2^4)/3.58}$$

$$= 1 - \frac{3.58}{27.33}\left[\frac{T_1^4 - T_3^4}{T_1^4 - T_2^4}\right]$$

$$= 1 - 0.131\left[\frac{T_1^4 - 0.48\,(T_1^4 + 1.08\,T_2^4)}{T_1^4 - T_2^4}\right]$$

$$= 1 - 0.131\left[\frac{T_1^4 - 0.48\,T_1^4 + 0.52\,T_2^4}{T_1^4 - T_2^4}\right]$$

$$= 1 - 0.131\left[\frac{0.52\,(T_1^4 - T_2^4}{(T_1^4 - T_2^4)}\right]$$

$= 1 - 0.131 \times 0.52 =$ **0.932 or 93.2%. (Ans.)**

HIGHLIGHTS

1. Heat transfer may be defined as the transmission of energy from one region to another as a result of temperature gradient and it takes place by three modes : conduction, convection and radiation.
2. Fourier's law of conduction : $Q = -\,kA\,\frac{dt}{dx}$

   where, Q = Heat flow through a body per unit time,

   A = Surface area of heat flow (perpendicular to the direction of flow),

   dt = Temperature difference,

dx = Thickness of body in the direction of flow, and

k = Thermal conductivity of the body.

3. Conduction of heat flow through slabs :

$$Q = \frac{A(t_1 - t_2)}{\left(\frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3}\right)} \quad \text{......3 slabs.}$$

4. Conduction of heat flow through pipe walls and lagging :

$$Q = \frac{(t_1 - t_2)}{\frac{1}{2\pi l}\left[\frac{1}{k}\log_e \frac{r_2}{r_1}\right]}.$$

5. Conduction of heat through a hollow sphere :

$$Q = \frac{t_1 - t_2}{\left(\frac{r_2 - r_1}{4\pi k r_1 r_2}\right)}.$$

6. Heat transfer by convection :

$$Q = hA\,(t_1 - t_2).$$

7. Overall heat transfer co-efficient :

$$U = \frac{1}{\frac{1}{h_{hf}} + \sum \frac{L}{k} + \frac{1}{h_{cf}}}.$$

8. Heat transfer between two fluids separated by the walls of a composite tube of solid material :

$$Q = \frac{(t_{hf} - t_{cf})}{\frac{1}{2\pi L}\left[\frac{1}{r_1 h_{hf}} + \frac{1}{k_1}\log_e \frac{r_2}{r_1} + \frac{1}{k_2}\log_e \frac{r_3}{r_2} + \frac{1}{r_4 h_{cf}}\right]}.$$

9. A heat exchanger may be defined as an equipment which transfers the energy from a hot fluid to a cold fluid, with maximum rate and minimum investment and running cost.

10. The net heat transfer in case of grey bodies with emissivities ε_1 and ε_2 is given by :

$$Q = \frac{\sigma A(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2}\right) - 1}$$

In case of concentric or long co-axial cylinder,

$$Q = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}.$$

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. The Fourier's law of heat transfer by conduction is expressed as

(a) $Q = kA^2 \frac{dt}{dx}$ (b) $Q = kA \frac{dt}{dx}$

(c) $Q = k^2A \frac{dx}{dt}$ (d) $Q = k^3A \frac{dx}{dt}$.

2. The heat transfer is constant when

(a) temperature remains constant with time (b) temperature decreases with time

(c) temperature increases with time (d) any of these.

3. The co-efficient of thermal conductivity is defined as

(a) Quantity of heat transfer per unit area per one degree drop in temperature

(b) Quantity of heat transfer per one degree temperature drop per unit area

(c) Quantity of heat transfer per unit time per unit area

(d) Quantity of heat transfer per unit time per unit area per one degree temperature drop per unit length.

4. The thermal conductivity is expressed as

(a) W/mK (b) W/m^2K

(c) W/hmK (d) W/h^2m^2K.

5. Heat transfer from higher temperature to low temperature takes place according to

(a) Fourier law (b) First law of thermodynamics

(c) Second law of thermodynamics (d) Zeroth law of thermodynamics.

6. Conduction through flat composite wall is given by :

(a) $Q = \dfrac{t_1 - t_4}{\dfrac{x_1}{k_1A} + \dfrac{x_2}{k_2A} + \dfrac{x_3}{k_3A}}$ (b) $Q = \dfrac{t_1 - t_4}{\dfrac{k_1A}{x_1} + \dfrac{k_2A}{x_2} + \dfrac{k_3A}{x_3}}$

(c) $Q = \dfrac{(t_1 - t_4)A}{\dfrac{k_1}{x_1} + \dfrac{k_2}{x_2} + \dfrac{k_3}{x_3}}$ (d) $Q = \dfrac{\dfrac{k_1A}{x_1} + \dfrac{k_2A}{x_2} + \dfrac{k_3A}{x_3}}{(t_1 - t_4)}$

where Q = heat transfer, t_1, t_2, t_3 and t_4 temperatures on surfaces of composite wall, x_1, x_2, x_3, x_4 thicknesses of different composite wall layers.

7. Conduction through hollow, radial one dimensional heat transfer is expressed as

(a) $Q = \dfrac{2\pi L(t_1 - t_2)k}{\log_e r_2/r_1}$ (b) $Q = \dfrac{2\pi L(t_1 - t_2)}{k(r_2 - r_1)}$

(c) $Q = \dfrac{2\pi L \log_e (t_1/t_2)}{(r_2 - r_1)k}$ (d) $Q = \dfrac{2\pi L(t_1 - t_2)k}{\log_e r_2/r_1}$

8. The radial heat transfer rate through hollow cylinder increases as the ratio of outer radius to inner radius

(a) decreases (b) increases

(c) constant (d) none of the above.

9. Stefan-Boltzmann law is expressed as

(a) $Q = \sigma AT^4$ (b) $Q = \sigma A^2T^4$

(c) $Q = \sigma AT^2$ (d) $Q = AT^4$.

10. The quantity of heat radiation is dependent on

(a) area of the body only (b) shape of the body only

(c) temperature of the body only (d) on all (a), (b) and (c).

ANSWERS

1. (b)	2. (a)	3. (d)	4. (a)	5. (c)	6. (a)	7. (a)
8. (a)	9. (a)	10. (c).				

THEORETICAL QUESTIONS

1. Enumerate the three modes by which heat can be transferred from one place to another. Which is the slowest of all ?
2. How do you define the thermal conductivity of a material ?
3. What do you understand by the terms 'convective heat transfer co-efficient' and 'overall heat transfer co-efficient'.
4. Derive an expression for heat loss in kJ/m²-hr through a composite wall of layers (*i*) without considering convective heat transfer co-efficients and (*ii*) considering the convective heat transfer co-efficients.
5. Classify the heat exchangers according to the flow directions of fluid and give few examples of each in actual field of application.
6. Prove that the mean temperature difference in a parallel-flow heat exchanger is given by

$$LMTD\ (t_m) = \frac{t_1 - t_2}{\log_e \frac{t_1}{t_2}}.$$

UNSOLVED EXAMPLES

1. The inner surface of a plane brick wall is at 40°C and the outer surface is at 20°C. Calculate the rate of heat transfer per m² of surface area of the wall, which is 250 mm thick. The thermal conductivity of the brick is 0.52 W/mK. [**Ans.** 41.6 W/m²]
2. Determine the rate of heat flow through the boiler wall made of 2 cm thick steel and covered with an insulating material of 0.5 cm thick. The temperatures at the inner and outer surfaces of the wall are 300°C and 50°C respectively.

   k (steel) = 58 W/mK

   k (insulation) = 0.116 W/mK. [**Ans.** 5.8 kW/m²]
3. A mild steel tank of wall thickness 10 mm contains water at 90°C. Calculate the rate of heat loss per m² of tank surface area when the atmospheric temperature is 15°C. The thermal conductivity of mild steel is 50 W/mK, and the heat transfer co-efficients for inside and outside the tank are 2800 and 11 W/m² K, respectively. Calculate also the temperature of the outside surface of the tank. [**Ans.** 820 W/m², 89.6°C]
4. A cold storage room has walls made of 0.23 m of brick on the outside, 0.08 m of plastic foam, and finally 15 mm of wood on the inside. The outside and inside air temperatures are 22°C and – 2°C respectively. If the inside and outside heat transfer co-efficients are respectively 29 and 12 W/m² K and the thermal conductivities of brick, foam and wood are 0.98, 0.02 and 0.17 W/mK respectively determine (*i*) the rate of heat removal by refrigeration if the total wall area is 90 m², and (*ii*) the temperature of the inside surface of the brick. [**Ans.** (*i*) 486.4 W, (*ii*) 20.28°C]
5. The wall of a refrigerated van is of 1.5 mm of steel sheet at outer surface, 10 mm plywood at the inner surface and 2 cm of glasswool in between. Calculate the rate of heat flow, if the temperatures of the inside and outside surfaces are – 15°C and 24°C.

   Take : k (steel) = 23.2 W/mK, k (glass-wool) = 0.014 W/mK

   and k (plywood) = 0.052 W/mK. [**Ans.** 6 kW/m²]
6. Sheets of brass and steel, each 10 mm thick, are placed in contact. The outer surface of brass is kept at 100°C and outer surface of steel is kept at 0°C. What is the temperature of the common interface ? The thermal conductivities of brass and steel are in the ratio of 2 : 1. [**Ans.** 66.7°C]
7. The wall of a furnace is made up of 250 mm of fire brick, k = 1.05 W/mK ; 120 mm of insulation brick, k = 0.85 W/mK, and 200 mm of red brick, k = 0.85 W/mK. The inner and outer surface temperatures of the walls are 850°C and 65°C respectively. Calculate the temperatures at the contact surfaces.

   Neglect the resistance of mortar joints. [**Ans.** 703°C, 210°C]

8. Calculate the heat flowing through a furnace wall 0.23 m thick, the inside and outside surface temperatures of which are 1000°C and 200°C respectively. Assume that the mean thermal conductivity of the wall material is 1.1 W/mK. Assuming that 7 mm of insulation (k = 0.075 W/mK) is added to the outside surface of the wall and reduces the heat loss 20% ; calculate the outside surface temperature of the wall. If the cost of the insulation is Rs. 70 per sq m what time will be required to pay for the insulation ? Base the calculations on the 24 hours operation per day and 199 days per year. Heat energy may be valued at Rs. 10 per 1000 kWh. [**Ans.** 3826 W/h-m^2 ; 74.3°C ; 1.916 years]

9. A flat wall of a furnace is composed of two layers of different materials having thicknesses of 0.115 m and 0.6 m with thermal conductivities of 0.16 W/m K and 10.6 W/m K respectively. If 1 kW/h of heat passes through every sq m area, estimate the drop in temperature at the contact between the two walls. The temperature inside the furnace is 1000°C and that at outside layer is 150°C. [**Ans.** 74°C]

10. A furnace wall consists of 250 mm fire brick, 125 mm insulating brick, and 250 mm building brick. The inside wall is at temperature of 600°C and the atmospheric temperature is 20°C. Calculate the heat loss per m^2 of wall area and the temperature of the outside wall surface of the furnace. The heat transfer co-efficient for the outside surface is 10 W/m^2 K, and the thermal conductivities of the fire brick, insulating brick and building brick are 1.4, 0.2 and 0.7 W/m K respectively.
Neglect radiation. [**Ans.** 0.46 kW/m^2 ; 66°C]

11. Hot air at a temperature of 60°C is flowing through a steel pipe of 100 mm diameter. The pipe is covered with two layers of different insulating materials of thicknesses 50 mm and 30 mm, and their corresponding thermal conductivities are 0.23 and 0.37 W/m K. The inside and outside heat transfer co-efficients are 58 and 12 W/m^2 K. The atmosphere is at 25°C. Find the rate of heat loss from a 50 m length of pipe. Neglect the resistance of the steel pipe. [**Ans.** 2.334 kW]

12. A steel pipe of 100 mm bore and 7 mm wall thickness, carrying steam at 260°C, is insulated with 40 mm of a high temperature diatomaceous earth covering. This covering is in turn insulated with 60 mm of asbestos felt. If the atmospheric temperature is 15°C, calculate the rate at which heat is lost by the steam per m length of the pipe. The heat transfer co-efficients for the inside and outside surfaces are 550 and 15 W/m^2 K, respectively and the thermal conductivities of steel, diatomaceous earth and asbestos felt are 50, 10.09 and 0.07 W/m K respectively. Calculate also the temperature of the outside surface.
[**Ans.** 116 W ; 22.8°C]

13. A 250 mm steam main, 225 metres long is covered with 50 mm of high temperature insulation (k = 0.095 W/m K) and 40 mm of low temperature insulation (k = 0.065 W/m K). The inner and outer surface temperatures as measured are 400°C and 50°C respectively. Calculate :
(*i*) The total heat loss per hour.
(*ii*) The total heat loss per m^2 of outer surface.
(*iii*) The heat loss per m^2 of pipe surface.
(*iv*) The temperature between the two layers of insulation.
Neglect heat conduction through pipe material.
[**Ans.** (*i*) 265514 kJ/h, (*ii*) 873.5 kJ/h, (*iii*) 1502.5 kJ/h, (*iv*) 215°C]

14. A steam pipe of 160 mm inside diameter and 170 mm outside diameter (k = 58 W/m K) is covered with first layer of insulating material of 30 mm thickness (k = 0.17 W/m K) and second layer of insulating material of 50 mm thickness (k = 0.093 W/m K). The temperature of steam passing through the pipe is 300°C and ambient air temperature surrounding the pipe is 30°C. Taking inner and outer heat transfer co-efficients 30 and 5.8 W/m^2 K respectively, find the heat lost per metre length of pipe.
[**Ans.** 216 W/m]

15. A small hemispherical oven is built of an inner layer of insulating fire brick 125 mm thick, and an outer covering of 85% magnesia 40 mm thick. The inner surface of the oven is at 800°C and the heat transfer co-efficient for the outside surface is 10 W/m^2 K, the room temperature is 20°C. Calculate the heat loss through the hemisphere if the inside radius is 0.6 m. Take the thermal conductivities of fire brick and 85% magnesia as 0.31 and 0.05 W/mK, respectively. [**Ans.** 1.93 kW]

16. A spherical shaped vessel of 1.2 m diameter is 100 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is 200°C. Thermal conductivity of the material is 0.3 kJ/m-h-°C. [**Ans.** 2262 kJ/h]

17. Exhaust gases flowing through a tubular heat exchanger at the rate of 0.3 kg/s are cooled from 400°C to 120°C by water initially at 10°C. The specific heat of exhaust gases and water may be taken as 1.13 and 4.19 kJ/kg K respectively, and overall heat transfer co-efficient from gases to water is 140 W/m^2 K. Calculate the surface area required when the cooling water flow is 0.4 kg/s.

(*i*) For parallel-flow ; (*ii*) For counter-flow. [**Ans.** (*i*) 4.0 m^2, (*ii*) 3.37 m^2]

18. Water flows inside a tube 50 mm in diameter and 3 m long at a velocity of 0.8 m/s. Determine the heat transfer co-efficient and the rate of heat transfer if the mean water temperature is 50°C and the wall is isothermal at 70°C. For water at 60°C, take k = 0.66 W/m K, ν (kinematic viscosity) = 0.478 × 10^{-6} m^2/s, and Prandtl number = 2.98. [**Ans.** 4075 W/m^2K ; 38.39 kW]

19. Liquid air at – 153°C is stored in the space of two concentric spheres of 21 cm and 30 cm diameters. The surface emissivities are 0.03. Assume the outer surface temperature is 27°C. Considering only radiation heat transfer and taking the latent heat of liquid air of 209 kJ/kg, find the rate of evaporation. Take $\sigma = 2.04 \times 10^{-4}$ kJ/h-m^2 K^4. [**Ans.** 21.7 kg/h]

20. A body at 1100°C in black surroundings at 550°C has an emissivity of 0.4 at 1100°C and an emissivity of 0.7 at 550°C. Calculate the ratio of heat loss by radiation per m^2,

(*i*) when the body is assumed to be grey with ε = 0.4

(*ii*) when the body is not grey. [**Ans.** (*i*) 70.22 kW, (*ii*) 62.42 kW]

21. A long steel rod, 20 mm in diameter, is to be heated from 427°C to 538°C. It is placed concentrically in a long cylindrical furnace which has an inside diameter of 160 mm. The inner surface of the furnace is at a temperature of 1093°C, and has an emissivity of 0.85. If the surface of the rod has an emissivity of 0.6, find the time required for the heating operation.

Take for steel : ρ = 7845 kg/m^3 and c = 0.67 kJ/kg K. [**Ans.** 29.88 s]